Vertical conflict of interest and horizontal inequities

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Abstract

We analyze a cheap talk game, à la Crawford and Sobel, in a two dimensional framework, with uniform prior, quadratic preferences and binary signaling rule. Credible information is revealed from the Sender to the Receiver when the conflict of interest vanishes through the alternative issues. The literature has focused on symmetrical equilibria and their sustainability upon limited exogenous asymmetry in preferences. We exhibit a second type of equilibrium, with endogenous asymmetry with respect to the revealed information. This type of equilibrium occurs with or without conflict of interest between the players, and is introduced by the multi-dimensionality. However, the conflict of interest conditions the design of decisions and their intrinsic meaning. Finally, we derive the existence of an influential equilibria for any conflict of interest.

Keywords: cheap talk, asymmetric information, inequity

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1 Introduction

As a matter of fact, conflict of interest is prevalent among most economical activities: employees and managers conflict on the optimal effort of the employees, managers and firm’s holders conflict on the optimal capital allocation, government and firms conflict on government’s policies (hence the existence of lobbyists) and on firm’s objectives (hence the existence of regulators), and so on. With this in consideration, Barnard (1938)\footnote{Cited by Laffont and Martimort (2002, p. 12).} already distinguished two ways to still allow for things to be done:
An organization can secure the efforts necessary to its existence, then, either by the objective inducements it provides or by changing states of mind. [...] We shall call the processes of offering objective incentives “the method of incentives”; and the processes of changing subjective attitudes “the method of persuasion”.

Indeed, although monetary transfers are central to design incentives in organizations, a significant number of decisions does not involve direct costs or benefits. In many situations, information is the core of the decision-making process.

In this perspective, the seminal work of Crawford and Sobel (1982) provides guidelines for the strategical aspect of information transmission between economical agents. Crawford and Sobel (1982) studies the effect of the conflict of interest on the influence an informed agent (the Sender, he) might have on the actions of an uninformed agent (the Receiver, she). The Sender possesses private information on a state of the world. He signals his information to the Receiver, who takes an action consequently. The conflict of interest consists of a difference in the preferences of the players concerning the Receiver’s action. Since the Sender’s signal results from his anticipation of the Receiver’s processing of the disclosed information, the conflict of interest gives reasons to the Receiver to be skeptical about the Sender’s signal. Therefore, the Sender’s signal may or may not be influential. Crawford and Sobel (1982) show that multiple influential equilibria are associated to a limited degree of conflict of interest. The equilibria are differentiated by their informativeness, and consequently the accuracy of the influence. An increased conflict diminishes both the number of equilibria and their informativeness. High degree of conflict precludes the possibility of influence.

The model of Crawford and Sobel (1982) restricts to a unique dimension for the private information of the Sender and the action of the Receiver. However, most interactions in real life are multi-dimensional. In this paper we study an extension of the model to two dimensions for the state of the world and the action, and we analyze the effect of this multi-dimensionality on the possibility of influence from the Sender to the Receiver, with respect to their conflict of interest. To this end, we restrict our attention to binary signaling rules ruling out the multiplicity of equilibria of the one dimensional model. This permits us to isolate the effect of adding a dimension on the design of the signals.

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2Many informative decisions are binary, for instance: endorsement, labeling, hiring, promotions, voting, court decisions. Furthermore, as the bit is the elementary element of information, one could consider to decompose more complex decisions into multiple binary decisions.
When extending the model to multiple dimensions for the private information and the action, results differ from the one dimensional case. As we will see, the multiplicity of dimensions permits any conflict of interest to “sneak” through the dimensions, and thereby to become an influential factor of decisions and actions of economical agents.

To support the intuition, consider a manager who has to assign two tasks to two employees. Each employee has an ability concerning the tasks. Assume that the manager is informed on the employees’ abilities, but the employees are not. For instance, the manager has higher expertise on the tasks to be done. The manager assigns the tasks upon his private information on the abilities. Employees infer their ability upon their observation of the assignment. Then, they exert effort on the task they are assigned to. They prefer an effort level as close as possible to their perceived ability. However, the manager’s assignment of the tasks results from the anticipation of the exerted efforts, upon which the manager has preferences that conflict with those of the employees. For instance, the manager might prefer higher exerted effort for both employees. Or he could have exogenous personal interest or disinterest concerning the efforts of the employees in the task they are assigned, and prefers higher exerted effort for an employee in a task, and lower exerted effort for the other employee in the other task. Given such a conflict of interest, we address the possibility for the manager to influence the employees to exert effort toward his preferred levels.

For instance, if the manager prefers the employees to exert higher effort than they do in both tasks (he has a positive bias in both dimensions), then, in equilibrium, the assignment rule cannot credibly disclose between high or low ability for both employees. Indeed, the manager would always prefer the employees to perceive an high ability, and exert high effort consequently. The assignment would always want to reveal the same information, and would consequently loose its informativeness.

Instead, as shown by Chakraborty and Harbaugh (2007) under general conditions, comparative statements are credible and influential in equilibrium. Indeed, when the manager bases the assignment on the comparison of abilities, then an employee perceives an high ability and the other a low ability. Relative to the manager’s preferences, the subsequent gain in effort of the former employee is counterbalanced by the loss in effort of the later. In particular the

\[ \text{Chakraborty and Harbaugh (2007)} \]

They also could be derived in a multi-task environment (Dewatripont et al., 2000). The effort exerted in the assigned task being subtracted to a fixed total extent of effort an employee exerts in his work.
conflict of interest vanishes through such an assignment rule.

Our contribution to the literature stands in three points. First, we note that the above mechanism extends to an arbitrary direction for the conflict of interest. Whatever the extent of conflicts of interest in the two dimensions, the credibility of the Sender’s decision is not compromised if the conflict is symmetrically balanced with respect to the corresponding anticipated/subsequent Receivers’ actions. This condition rules out the effect of the conflict of interest on communication. Under our specific setting, this generalizes the main result of Chakraborty and Harbaugh (2007) to any direction.

For instance, when comparing the employees’ abilities as above, the conflict of interest vanishes within the “dimension of comparison”, supported by the differences of the employees’ abilities. In general, the conflict of interest vanishes on the dimension orthogonal to its direction, provided that the Receiver’s inferences are symmetrical with respect to this dimension. We obtain thus a symmetrical equilibrium when the symmetry of the prior fits with the symmetry of the conflict of interest. However, it is worth noting that upon such a symmetrical equilibrium, the conflict of interest has no readable effect on the Receiver’s action – the same decision-making rule could arise in equilibrium without conflict of interest.

Second, we exhibit another type of equilibrium that might occur beside the symmetrical equilibria. The second type of equilibrium is asymmetrical with respect to the revealed information, and the asymmetry might occur endogenously, that is despite a symmetrical conflict of interest.

The multi-dimensionality introduces a new multiplicity of equilibria, where information is revealed along combinations of the dimensions. Suitable combinations limits the extent of conflict of interest in the dimension of revealed information, and permits influential equilibrium to occur. Following the result of Crawford and Sobel (1982), such influential equilibrium occurs even if the conflict does not fully vanishes on the (unique) dimension of the revealed information. For instance, limited asymmetry of the conflict does not rule out communication, but permits influential equilibrium to occur as a limited asymmetrical equilibrium. We show that an asymmetrical equilibrium might also occur with a symmetrical conflict, introducing endogenous asymmetry in the revealed information. As mentioned above, a symmetrical decision-making rule associated with symmetrical bias fully vanishes the conflict of interest within the dimension of revealed information. An asymmetrical decision-making rule associated with symmetrical bias limits the extent of conflict of interest without vanishing it, and maintains the communi-
cation. For instance, in the organization, despite a symmetrical conflict of interest (and even without conflict of interest), the manager could assign asymmetrically the tasks to employees, introducing inequity.

Third, the above multiplicity of equilibria permits us to derive the existence of an influential equilibrium whatever the direction and the extent of the conflicts of interest in the two dimensions. In particular, in organizations, conflict of interest possibly impacts decisions and their perception, whatever the direction and extent of conflict.

Extensions of the model of Crawford and Sobel (1982) to multiple dimensions have already been investigated by the literature. Most studies concern either the possibility of full revelation, or binary signaling rules. Battaglini (2002) shows that when the state space is the full Euclidean space in two dimensions, the Sender and the Receiver have a dimension of common interest. The author derives the possibility of a fully revealing equilibrium in a game with two senders and one receiver. In contrast, Ambrus and Takahashi (2008) show that there is no fully revealing equilibrium when the state space is restricted and bias of the two senders are large enough and are not in similar directions. In our setting, we do not focus on full revelation, but on the possibility of influence through binary decisions from the Sender. This is necessarily not fully revealing when the state space is infinite.

Also focusing on the possibility of influence, symmetrical comparative statements have been shown to be sustainable in equilibrium by Chakraborty and Harbaugh (2007), provided a symmetrical prior and a symmetrical bias with respect to the dimensions, and a supermodular condition on the utility functions. In contrast, Levy and Razin (2007) show that there is no possibility of influence upon large asymmetries in the game. We note that the result of Chakraborty and Harbaugh (2007) can be generalized by symmetry to any direction, provided that the symmetrical conditions are fulfilled with respect to that direction. Upon symmetrical comparative equilibria, the revealed information is conditioned by the direction of the conflict, and not its scale. In particular, the same information could be revealed without conflict of interest.

The asymmetrical equilibria we exhibit have not been investigated in the literature upon

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4An exception is the work of Jäger et al. (2011), who study the possible quantizations of the set of states through messages. They characterize the equilibria as so-called Voronoi partitions of the set of states. However, their main concern rules out the conflict of interest of the players.
general conditions on the prior, conflict of interest or utility functions.\footnote{The existence of other equilibria does not permit to derive such an existence from classical fixed point theorems for instance. Stronger topological theorems, if any, would be needed. For instance,\cite{Chakraborty2010} use the Borsuk-Ulam Theorem to derive the existence of an influential equilibrium whatever is the prior, provided that the Sender’s preferences does not depend on the state.} Under a different setting,\cite{Kamphorst2016} exhibit endogenous asymmetrical equilibria as we do, assuming quadratic preferences for the Receiver, linear preferences for the Sender, uniform prior and binary signaling rule.\footnote{\cite{Crutzen2013} show that under this setting, the number of messages cannot exceed three.} Linear preferences for the manager implies that he always prefers the employees to exert the highest possible effort. Our setting explores any fixed difference for preferences of the players. The authors interprets the existence of such an asymmetrical equilibrium as representing a possible discriminatory practice from the players. We might derive from our result that the discrimination considered by\cite{Kamphorst2016} is precisely the consequence of the multi-dimensionality, which allows different degree of informativeness in both dimensions. It might occur without conflict of interest, but is not ruled out by the conflict. Instead, the conflict impacts the design of the decisions and their interpretation by the Receiver, so that its effect is to condition the intrinsic meaning of the Sender’s decisions.

The next section gives the model. Section 3 derives the analyze and results. Sections 3.1 and 3.2 study the symmetric and asymmetric equilibria respectively, associated with specific families of bias. Section 3.3 derive all the equilibria of the game. Section 4 is a concluding section. Proofs are given in Appendix.

## 2 Model setup

We consider an agent, henceforth called the Sender ($S$, he), who is informed about a state of the world $\theta$, and an agent, henceforth called the Receiver ($R$, she), who is not. The state of the world $\theta = (\theta_1, \theta_2)$ is the realization of a uniform random variable over a compact convex set $\Theta \subset \mathbb{R}^2$ normalized to $[0, 1]^2$. The Sender makes a decision $D$, among two alternatives $D_1$ and $D_2$, upon his observation of the state. Then, the Receiver observes the Sender’s decision, and takes an action (or effort) $a \in \mathbb{R}^2$ consequently.\footnote{The multidimensionality of the game permits also to report on two Receivers with a one dimensional action $a$, as well as two Senders who each observes one of the $\theta_i$ and who agree on the decision, which is then publicly observed by the Receiver(s).} Preferences of both players rely on the state...
of the world $\theta$ and on the Receiver’s action $a$, but not on the decision $D$. The decision is purely informative. The conflict of interest between the players occurs as a difference in preferences concerning the Receiver’s action $a$. The Receiver prefers her action to be as close as possible to the state of the world. Her utility $U^R$ decreases with the (Euclidean) distance between $a$ and $\theta$. We set

$$U^R(a, \theta) = -\|a - \theta\|^2.$$  

The Sender prefers the Receiver’s action to be as close as possible to a shifting of the state of the world. The Sender’s utility $U^S$ decreases with the distance between $a$ and $\theta + b$, with $b \in \mathbb{R}^2$ representing the conflict of interest. We set

$$U^S_b(a, \theta) = -\|a - (\theta + b)\|^2.$$  

The timing of the game is as follows:

1. Nature draws the state of the world $\theta = (\theta_1, \theta_2)$, and reveals it to the Sender, but not to the Receiver (she has a uniform prior);
2. the Sender makes a decision $D \in \{D_1, D_2\}$ upon his observation of $\theta$;
3. the Receiver observes the Sender’s decision and update her belief about the state of the world;
4. the Receiver chooses her action $a \in \mathbb{R}^2$ according to her posterior belief;
5. payoffs are realized.

We look for perfect Bayesian equilibria of this game, that is: (i) Receiver’s action strategy is optimal, given her belief about the state of the world; (ii) the Sender’s decision strategy is optimal, given the Receiver’s action strategy and belief; (iii) belief is updated according to Bayes’ rule.

3 Analyze

The strategies of the players are as follows. The Sender makes a binary decision $D(\theta) \in \{D_1, D_2\}$ upon his observation of the state of the world $\theta$. Let $D_i^{-1}$ be the set of $\theta$ for which
decision $D_i$ is made. If $D_i^{-1} \neq \emptyset$, upon her observation of $D_i$, the Receiver’s takes the action that maximizes her expected utility, at

$$a_i = a(D_i) = \arg \max_a \int_{\theta \in D_i^{-1}} U^R(a, \theta) \, d\theta = E[\theta | D(\theta) = D_i].$$

(1)

Reciprocally, in equilibrium, the Sender makes the decision that maximizes his utility upon his observation of $\theta$:

$$D(\theta) = \arg \max_{D \in \{D_1, D_2\}} U^S_b(a(D), \theta) = \{D_i, U^S_b(a(D_i), \theta) \geq U^S_b(a(D_{-i}), \theta)\},$$

(2)

where $-i$ represents the element of $\{1, 2\} \setminus \{i\}$.

Note that there is always a babbling equilibrium, upon which no information is revealed. It occurs when the Sender always makes the same decision $D_i$, for $a_i \in \{1, 2\}$, whatever $\theta$, so that $D_i^{-1} = \Theta$ and $D_{-i}^{-1} = \emptyset$, and the Receiver always takes the same action, consequently at $a = E[\theta | \theta \in \Theta] = E[\theta] = (\frac{1}{2}, \frac{1}{2})$.

### 3.1 Symmetrical equilibria

Chakraborty and Harbaugh (2007) show that whatever the conflict of interest, if conflict of interest and prior are both symmetrical across the two dimensions, then deciding upon the comparison between the dimensions is credible and influential.

Basically, the symmetry of the conflict of interest and of the prior implies that independently of the observed state of the world $\theta = (\theta_1, \theta_2)$, when ranking, say $\theta_1$ over $\theta_2$, the Sender’s gain or loss (relative to his conflict) from the Receiver’s inference of a superior $\theta_1$ is exactly counterbalanced by his loss or gain in the Receiver’s inference of an inferior $\theta_2$. Therefore, the conflict of interest is annihilated through the ranking of $\theta_1$ and $\theta_2$. The Sender’s alternative decisions

$$D(\theta) = \begin{cases} D_1, & \text{if } \theta_1 \geq \theta_2, \\ D_2, & \text{if } \theta_1 < \theta_2, \end{cases}$$

are then credible and influential in equilibrium.

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*Chakraborty and Harbaugh* (2007) show that comparative statements are credible and influential in equilibrium in the general setting of a symmetric prior distribution for the state $\theta$, and some symmetrical supermodularity conditions for the utility functions of the players. In contrast, *Levy and Razin* (2007) show that substantial asymmetry in the prior precludes the possibility of influence when the conflict of interest is large enough.
Let us first note that the condition of existence of such a symmetrical equilibrium extends to any direction for the conflict of interest. The condition in equilibrium is that the prior must be symmetrical with respect to the direction of the conflict, so that the alternative issues induce symmetrical actions with respect to this direction. Therefore, the conflict annihilates across the alternative actions. Reciprocally, any axis of symmetry for the prior defines a decision-making rule for a Sender whose bias’ direction is aligned with that of the axis of symmetry. Upon uniform prior, we obtain the following proposition.

**Proposition 1.** Conditional on his bias $b$, a sender might decide upon the following decision-making rule:

- a comparative symmetrical decision-making rule, denoted $C$:

$$
D(\theta) = \begin{cases} 
D_1, & \text{if } \theta_1 \geq \theta_2, \\
D_2, & \text{if } \theta_1 < \theta_2, 
\end{cases}
$$

if the bias $b = (b, b), b \in \mathbb{R}$, is symmetrical with respect to the dimensions;

- an aggregative symmetrical decision-making rule, denoted $A$:

$$
D(\theta) = \begin{cases} 
D_1, & \text{if } \theta_1 + \theta_2 \geq 1, \\
D_2, & \text{if } \theta_1 + \theta_2 < 1, 
\end{cases}
$$

if the bias $b = (b, -b), b \in \mathbb{R}$, is opposed with respect to the dimensions;

- an half-babbling decision-making rule, denoted $H_i$, $i \in \{1, 2\}$:

$$
D(\theta) = \begin{cases} 
D_1, & \text{if } \theta_i \geq \frac{1}{2}, \\
D_2, & \text{if } \theta_i < \frac{1}{2}, 
\end{cases}
$$

if the bias $b = (0, b)$ (when $i = 1$), or $b = (b, 0)$ (when $i = 2$), $b \in \mathbb{R}$, concerns only one of the two dimensions.

Chakraborty and Harbaugh (2007) derive the decision-making rule $C$ from more general conditions on the prior and preferences of the players (symmetrical prior, symmetrical bias, and a supermodularity condition of the utility functions). By symmetry, all decision-making rules exhibited in Proposition 1 could easily be derived under these general conditions. The key element is the symmetry of the parameters of the game considered ones with respect to
the others (prior, bias, and utility functions). Proposition\textsuperscript{1} permits to lighten the relationship between the different decision-making rules. For instance, note that when $b = (0, 0)$, each decision-making rule $C$, $A$ and $H_i$, $i = 1, 2$ occurs in equilibrium of the same game. In particular, the multi-dimensionality introduces a new multiplicity of equilibria for the fixed level of informativeness we consider, by our restriction to binary decisions. However, the extent of conflict of interest has no repercussion on decisions and subsequent actions of the Receiver upon the decision-making rules of Proposition\textsuperscript{1}. In particular, the same decision-making rules occur in equilibrium of the game with no bias.

Also, note that decision-making rules $H_i$, $i = 1, 2$, are asymmetrical with respect to the dimensions. Upon $H_i$, the revealed information concerns a unique dimension of the state. The Sender is babbling on the other dimension, where the bias can be arbitrary. However, if $b \neq 0$, $H_i$ is exogenously asymmetrical. In the next section, we will exhibit equilibria associated with positive conflict, where similar differences of informativeness among the dimensions are endogenously introduced by the multi-dimensionality.

Proposition\textsuperscript{1} also enlightens the fact that with respect to the conflict of interest, the factors upon which the decision is made are formally similar, despite being interpretatively different: upon $C$, the Sender compares the two states; upon $A$, he discloses between high and low total value, and upon $H_i$ he discloses between high and low total value of one of the component of the state. In particular, the direction of conflict of interest conditions the interpretation of the decision.

As an illustration, a manager’s assignment of the tasks influences the employees’ efforts in equilibrium if it is based on the comparison of abilities when the bias is symmetrical with respect to the dimensions; it could either be based on the sum of the abilities (hence on the team’s ability) when the components of the bias are opposed; and it could either be based on the ability of only one of the employees when there is no bias relative to that employee.

Let us now show that the properties of the equilibria of Proposition\textsuperscript{1} compare to those of a one dimensional model, within a suitable dimension.

For instance, when symmetrically comparing $\theta_1$ and $\theta_2$, the “dimension of comparison” is represented by $\theta_1 - \theta_2$. The comparison occurs in equilibrium precisely because the conflict vanishes through the alternative statements $\theta_1 - \theta_2 \geq 0$ and $\theta_1 - \theta_2 < 0$. But as in a one dimensional model, this type of equilibrium carries on as long as the conflict of interest remains limited when considered through the prism of the alternative statements. For instance, consider
the following decision-making rule, denoted \( C(c) \), for \( c \in (-1, 1) \):

\[
D(\theta) = \begin{cases} 
D_1, & \text{if } \theta_1 \geq \theta_2 + c \\
D_2, & \text{if } \theta_1 < \theta_2 + c.
\end{cases}
\]

Note that when \( |c| \to 1 \), \( C(c) \) tends to babble (either \( D_1 \) or \( D_2 \) is always made, whatever \( \theta \)).

The following proposition precisely establishes that comparison upon \( C(c) \) might occur in equilibrium, provided that the conflict of interest is limited within the “dimension of comparison”.

**Proposition 2.** Assume the conflict of interest is given by \( b = (b_1, b_2) \in \mathbb{R}^2 \), and set \( \Delta b = |b_2 - b_1| \), representing the exogenous asymmetry of the conflict across the dimensions. An influential equilibrium with \( C(c) \) as decision-making rule is sustainable if and only if \( \Delta b < \frac{1}{2} \).

The decision-making rule \( C(c) \) illustrates the deviation of the comparative symmetrical decision-making rule \( C \) that occurs with limited asymmetry in the bias, relative to the symmetry of the other aspects of the game. A similar result holds for the aggregative \( A \) and half-babbling \( H_i \) decision-making rules, through decision-making rules that we denote \( A(c) \) and \( H_i(c) \) respectively.

Figures 1, 2 and 3 illustrate the different decision-making rules \( C, A, H_i \) and their deviations \( C(c), A(c), H_i(c) \), due to limited asymmetry in the bias with respect to the alternative issues.

**Fig. 1:** Comparative decision-making rule \( C \), and its deviations from bias \( b = (b_1, b_1 + \Delta b) \)

**Fig. 2:** Aggregative symmetrical equilibrium \( A \) and its deviations from bias \( b = (-b_1, b_1 + \Delta b) \)
Fig. 3: Half babbling decision-making rules $\mathcal{H}_1$ and its deviations from bias $b = (b_1, 0 + \Delta b_1)$

### 3.2 Asymmetrical equilibria

The condition for the equilibria of the preceding section to occur is the limited extent of asymmetry of the conflict of interest with respect to a given symmetry of the prior. In this section we exhibit a different type of equilibria. These equilibria also deviate from the symmetrical equilibria, but even with symmetrical conflict of interest. Therefore, they introduce an endogenous asymmetry in the decisions.

Let us give the geometrical intuition. Consider an *asymmetrical* decision making rule

$$
\mathcal{D}(\theta) = \begin{cases} 
\mathcal{D}_1, & \text{if } \theta_1 \geq \varphi(\theta_2), \\
\mathcal{D}_2, & \text{if } \theta_1 < \varphi(\theta_2),
\end{cases}
$$

as illustrated in Figure 4, where $a_i = E[\theta | \theta \in \mathcal{D}^{-1}_i]$ denotes the Receiver’s action conditional on the decision $\mathcal{D}_i$. 

Fig. 4: An endogenous asymmetrical equilibrium

Assume the Sender observes the state $\theta = (\theta_1, \theta_2)$ drawn in Figure 4. In particular, $\theta \in D_{-1}^{-1}$, which means that given the rule (3), the Sender would decide $D_1$.

Assume for the moment that there is no conflict of interest. Then the Sender and the Receiver prefer the Receiver to act as close as possible to the observed value $\theta$. According to Figure 4, $a_2$ is closer to $\theta$ than $a_1$, so the Sender prefers the Receiver to infer $a_2$, and thus decides $D_2$. Therefore, the decision-making rule corresponding to the one represented in Figure 4 cannot occur in equilibrium.

Now assume a positive symmetrical conflict of interest $b = (b, b)$, $b > 0$, as represented in Figure 4. Then, the Sender prefers the Receiver to make the inference that is closer to $\theta + b$ instead. In that case, he does decide $D_1$, in accordance with the rule represented.

If a decision-making rule precisely identifies the set $D_1$ to the set of $\theta$ such that $\theta + b$ is closer to $a_1$ than to $a_2$, then it does occur in equilibrium. Figure 4 illustrates such an asymmetrical equilibrium. Note that it is asymmetrical despite the symmetry of all exogenous aspects of the game.

Proposition 3. Consider a symmetrical bias $b = (b, b), b \in \mathbb{R}$. If $|b| > \frac{9}{184}$, then the Sender might decide asymmetrically across the dimensions of the state of the world in equilibrium.

Figure 5 illustrates the various decision-making rules corresponding to the cases $b = (b, b), b > \frac{9}{184}$ (the cases $b < \frac{9}{184}$ are easily obtained from symmetry).
Fig. 5: Endogeneous asymmetrical decision-making rules, associated with bias $b = (b, b)$, $b > \frac{9}{184}$

As illustrated in Figure 5, the extent of conflict of interest impacts the design of the decisions. With small conflict of interest, an aggregative decision-making rule $\mathcal{A}(c)$ occurs in equilibrium. With $b$ increasing, it turns to a comparative decision-making rule $\mathcal{C}(c)$. In particular, while the multi-dimensionality adds a new multiplicity of equilibria without conflict of interest (as we noted, decision-making rules $\mathcal{A}$, $\mathcal{C}$, $\mathcal{H}_i$ all occur in equilibrium of the game with $b = (0,0)$), such a multiplicity is not ruled out in general with conflict of interest, whatever the extent of the conflict of interest. To each symmetrical bias $b = (b, b)$, the corresponding asymmetrical equilibrium results from two factors:

- there is enough flexibility in the one dimensional model for the existence of influential equilibrium despite conflict of interest, if limited;
- there is enough space in the multi-dimensional model for the existence of dimensions along which the conflict of interest is sufficiently limited.

The conflicts impact the design of the decision-making rules so that it remains limited through the alternative issues. From an interpretative point of view, the conflict conditions the intrinsic meaning of decisions in equilibrium.

For instance, with high symmetrical conflict of interest, multiple comparative equilibria are sustainable in equilibrium. A symmetrical comparison is sustainable through $\mathcal{C} = \mathcal{C}(0)$, as well as an asymmetrical comparison through $\mathcal{C}(c)$, $c \neq 0$ as illustrated in Figure 5 (with $b \to \infty$).
3.3 Class of games

In Propositions 1, 2 and 3 we have determined the decision-making rules associated with some particular bias $b$. In this section, we extend the results to arbitrary bias.

Let us denote $\Gamma_b$ the game associated with a conflict of interest $b \in \mathbb{R}^2$ between the Sender and the Receiver. Denote $\mathcal{E}(\Gamma_b)$ the set of decision-making rules that occurs in an influential equilibrium of $\Gamma_b$. We aim at showing that for any $b$, $\mathcal{E}(\Gamma_b)$ is non empty.

Given a decision making rule $D$, let $B(D) \subseteq \mathbb{R}^2$ be the set of conflicts of interest $b$ for which $D \in \mathcal{E}(\Gamma_b)$. The following lemma first derive $B(D)$ from a given $D \in \mathcal{E}(\Gamma_b)$.

**Lemma 1.** Let $b \in \mathbb{R}^2$, and let $D \in \mathcal{E}(\Gamma_b)$. Let $a_1$ and $a_2$, $a_1 \neq a_2$ be the associated actions of the Receiver. Then $B(D) = \{b' \in \mathbb{R}^2, (b' - b) \perp (a_1 - a_2)\}$.

Note that $(b' - b) \perp (a_1 - a_2) \iff b' \cdot (a_1 - a_2) = b \cdot (a_1 - a_2)$, so that Lemma 1 tells us that given a decision-making rule $D \in \mathcal{E}(\Gamma_b)$ and the corresponding actions $a_i$, $i \in \{1, 2\}$, the set of bias $b'$ for which $D$ occurs in equilibrium of $\Gamma_b$ are those $b'$ that project orthogonally as $b$ on the line $(a_1 a_2)$. Graphically (see Figure 6), note that for each $i \in \{1, 2\}$, the set $D_i^{-1}$ of states that lead to decision $D_i$ corresponds to the region situated on one side of the perpendicular bisector of the segment $a_i - b$ and $a_2 - b$ (it is a direct consequence of (2)). Therefore, $B(D)$ corresponds to bias $b'$ such that $a_1 - b'$ and $a_2 - b'$ induce the same perpendicular bisector.

Figure 6 represents an element $-b'$ of $-B(D)$, i.e. such that $b' \in B(D)$.

Now consider $\rho$ the quarter turn rotation of the plan (in the trigonometric sens) with center...
The center of $\Theta = [0, 1]^2$. The following lemma is a direct consequence of the invariance of $\Theta = [0, 1]^2$ through $\rho$.

**Lemma 2.** Let $\mathbf{b} = (b_1, b_2) \in \mathbb{R}^2$, and let $\mathcal{D} \in \mathcal{E}(\Gamma_b)$. Then the decision-making rule $\rho(\mathcal{D})$, defined by

$$\rho(\mathcal{D})(\mathbf{\theta}) = \begin{cases} 
\rho(\mathcal{D})_1, & \text{if } \rho^{-1}(\mathbf{\theta}) \in \mathcal{D}_1^{-1}, \\
\rho(\mathcal{D})_2, & \text{if } \rho^{-1}(\mathbf{\theta}) \in \mathcal{D}_2^{-1}, 
\end{cases}$$

if an element of $\mathcal{E}(\Gamma_{(-b_2,b_1)})$.

As an illustration, Figure 7 represents the decision-making rule $\rho(\mathcal{D})$, associated with bias $(-b,b)$ derived from the decision-making rule $\mathcal{D}$ of Figure 4 associated with bias $(b,b)$.

![Fig. 7: An equilibrium associated with the bias $\mathbf{b} = (-b,b)$](image)

Denote $\mathcal{D}^{as}(\mathbf{b})$ the asymmetrical decision-making rules exhibited in Proposition 3, associated with bias $\mathbf{b} = (b,b)$, $b > \frac{9}{184}$. By applying $\rho$ multiple times to $\mathcal{D}^{as}(\mathbf{b})$, Lemma 2 permits us to extend Proposition 3 to bias $\mathbf{b} = (b,b)$ with negative $b$, and bias $\mathbf{b} = (b,-b)$ with positive or negative $b$. In particular, for any $b > \frac{9}{184}$,

$$\rho(\mathcal{D}^{as}(\mathbf{b})) \in \mathcal{E}(\Gamma_{(-b,b)}), \quad \rho^2(\mathcal{D}^{as}(\mathbf{b})) \in \mathcal{E}(\Gamma_{(-b,-b)}), \quad \text{and } \rho^3(\mathcal{D}^{as}(\mathbf{b})) \in \mathcal{E}(\Gamma_{(b,-b)}).$$

Proposition 4 above is derived from the fact that the sets of $\mathcal{B}(\mathcal{D})$ for $\mathcal{D}$ ranging among the decision-making rules $\mathcal{C}(c)$, $\mathcal{A}(c)$, $\mathcal{H}_i(c)$, and $\mathcal{D}^{as}(\mathbf{b})$ and their rotations cover all possible bias $\mathbf{b} \in \mathbb{R}^2$.

**Proposition 4.** For any $\mathbf{b} \in \mathbb{R}^2$, the game $\Gamma_b$ possesses an influential equilibrium $\mathcal{D} \in \mathcal{E}(\Gamma_b)$. Moreover, either $\mathcal{D} \in \mathcal{E}(\Gamma_{(b,b)})$, or $\mathcal{D} \in \mathcal{E}(\Gamma_{(-b,b)})$ for some $b \in \mathbb{R}$. 

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Therefore, whatever the conflict of interest between the Sender and the Receiver, there is a ground for binary decisions of the Sender to be influential. In particular, in organizations, conflict of interest might navigate as an influential factor through decisions and actions.

4 Conclusion

We have determined all the equilibria of an extension of the model of Crawford and Sobel (1982) to two dimensions, assuming uniform prior, quadratic preferences, and a binary signaling rule. Beside the comparative symmetrical equilibria that was already exhibited in the literature, we have exhibited multiple continua of equilibria that rely on the symmetries of the parameters of the game, ones with respect to the others. Information could be asymmetrically disclosed despite a totally symmetrical game. The new multiplicity of equilibria results from the flexibility of the one-dimensional game with respect to the conflict of interest, combined with the multiplicity of dimensions along which information might be disclosed. The conflict of interest does not rule out the possibility of influence, but rather conditions the design of influential decisions and their perception.

References


Appendix

Proof of proposition [1]

Considering $C$, one derives from (1) the induced actions

$$ a_1 = \left( \frac{2}{3}, \frac{1}{3} \right), \quad \text{and} \quad a_2 = \left( \frac{1}{3}, \frac{2}{3} \right). $$

Upon $A$ (resp. $H_i$), we obtain $a_1 = \left( \frac{2}{3}, \frac{1}{3} \right)$ and $a_2 = \left( \frac{1}{3}, \frac{1}{3} \right)$ (resp. $a_1 = \left( \frac{2}{4}, \frac{1}{4} \right)$ and $a_2 = \left( \frac{1}{4}, \frac{1}{4} \right)$ when $i = 1$, or $a_1 = \left( \frac{1}{2}, \frac{3}{2} \right)$ and $a_2 = \left( \frac{1}{2}, \frac{1}{4} \right)$ when $i = 2$.

Reciprocally, given $a_1$ and $a_2$ and $b$, for instance $a_1 = \left( \frac{2}{3}, \frac{1}{3} \right)$, $a_2 = \left( \frac{1}{3}, \frac{2}{3} \right)$ and $b = (b, b)$,
then from (2), we have $D(\theta) = D_1$ if and only if (up to a null measure set)

$$-\|a_1 - (\theta + b)\|^2 \geq -\|a_2 - (\theta + b)\|^2$$

$$\iff -\left(\frac{2}{3} - (\theta_1 + b)\right)^2 - \left(\frac{1}{3} - (\theta_2 + b)\right)^2 \geq -\left(\frac{1}{3} - (\theta_1 + b)\right)^2 - \left(\frac{2}{3} - (\theta_2 + b)\right)^2$$

$$\iff \frac{1}{3} (1 - 2(\theta_2 + b)) - \frac{1}{3} (1 - 2(\theta_1 + b)) \geq 0$$

$$\iff \theta_1 \geq \theta_2$$

and similarly $D(\theta) = D_2$ if and only if $\theta_1 < \theta_2$, hence the corresponding decision-making rule.

The proofs for $A$ and $H_i, i = 1, 2$ are similar.

**Proof of proposition 2**

Consider a decision-making rule

$$D(\theta) = \begin{cases} 
D_1, & \text{if } \theta_1 \geq \theta_2 + c, \\
D_2, & \text{if } \theta_1 < \theta_2 + c,
\end{cases}$$

over $(\theta_1, \theta_2) \in [0, 1]^2$ for some $c < 0$. Then from (1) we obtain

$$a_1 = E[\theta | \theta_1 \geq \theta_2 + c] = \left(\frac{1}{3} c^3 + 3c^2 + 3c - 2, \frac{1}{3} c^3 + 3c - 1\right),$$

and

$$a_2 = E[\theta | \theta_1 < \theta_2 + c] = \left(\frac{c + 1}{3}, \frac{2 - c}{3}\right).$$

Reciprocally, from (2), decision $D_1$ is made if and only if

$$-\|a_1 - (\theta + b)\|^2 \geq -\|a_2 - (\theta + b)\|^2,$$

with $b = (b_1, b_2)$. Setting $b = (b_1, b_1 + \Delta b)$ with $\Delta b \in \mathbb{R}$, this gives in equilibrium

$$\theta_1 \geq \theta_2 + \frac{1}{3} \frac{c(c+2)(2c-1)}{c^2+2c-1} + \Delta b,$$

so that in equilibrium $c$ must solve

$$c = \frac{1}{3} \frac{c(c+2)(2c-1)}{c^2+2c-1} + \Delta b,$$

that is $\frac{1}{3} \frac{c^3+3c^2-c}{c^2+2c-1} = \Delta b$. This defines an increasing map $\Delta b \mapsto c(\Delta b)$ from $\Delta b \in (-\frac{1}{2}, 0)$ to $(-1, 0]$. A similar argument applies to the case $\Delta b \in [0, \frac{1}{2})$ with $c(\Delta b) \in [0, 1)$. 

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Proof of proposition \[^3\]

Let $D_1$ and $D_2$ be the two alternative Sender’s decisions, let $a_i = a(D_i)$, be the corresponding Receiver’s induced actions in equilibrium. For any $\theta \in \Theta$, the Sender decides $D_1$ for instance, if and only if

$$-||a_1 - (\theta + b)||^2 > -||a_2 - (\theta + b)||^2$$

$$\iff ||(a_1 - b) - \theta|| < ||(a_2 - b) - \theta||.$$

Thus, the Sender prefers to induce $a_1$ whenever $\theta$ is closer to $a'_1 = a_1 - b$ than to $a'_2 = a_2 - b$, and conversely. The set $\Theta_0$ of indifferent states for the Sender is the perpendicular bisector of $a'_1$ and $a'_2$ (see Figure 8 for an illustration).

![Fig. 8: A decision-making rule](image)

In equilibrium, the Sender’s decision is made upon the fact that $\theta$ belongs to the region situated on one side or the other of $\Theta_0$, and the Receiver’s actions $a_1$ and $a_2$ must correspond to the expected values of the $\theta$s in these regions.

For the determination of the possible equilibria, we have to consider sections of the square $\Theta = [0; 1]^2$ that may occur as perpendicular bisectors of the shifted actions $a_1 - b$ and $a_2 - b$. Note that it is sufficient to consider only the four possibilities (A), (B), (C) and (D) given in Figure 9. Indeed, since the bias is symmetrical with respect to the dimensions, an equilibrium holds in one with a decision-making rule corresponding to one of the cases (A), (B), (C) or (D) if and only if the corresponding symmetrical case (drawn in dashed line) does, where symmetry is relative to the first diagonal $\theta_1 = \theta_2$. The cases (A) to (D) and their symmetric covers all type of sections of the square.
Next we will compute the possible equilibria in each of the cases (A) to (D). The method is independent of the case:

- first, we start with a parametrization $\Theta_0(c, e)$ of $\Theta_0$ such that $\Theta_0$ is represented by the affine line passing through the points $(c, \frac{1}{2})$, $(c + e, 1)$ and $(c - e, 0)$, with $c, e \in \mathbb{R}$; write

$$\Theta_0(c, e) = \{ \bm{\theta} \in [0, 1]^2, \theta_1 = 2e\theta_2 + c - e \},$$

and the different cases (A), (B), (C) and (D) distinguish with associated conditions on $c$ and $e$;

- second, we compute the component $a_{ij}$ of action $a_i$ conditional on decision $D_j$ with respect to the parameters $c$ and $e$ defined in the first step. Component $a_{ij}(c, e) = E_f(\theta_i|\bm{\theta} \in D^{-1}_j)$ is given by the mean values of $\bm{\theta} = (\theta_1, \theta_2)$ over the set $D^{-1}_j$:

$$a_{11}(c, e) = \frac{\int_{\theta_1 > 2e\theta_2 + c - e} \theta_1 \, d\theta}{\int_{\theta_1 > 2e\theta_2 + c - e} 1 \, d\theta}, \quad a_{12}(c, e) = \frac{\int_{\theta_1 < 2e\theta_2 + c - e} \theta_1 \, d\theta}{\int_{\theta_1 < 2e\theta_2 + c - e} 1 \, d\theta}, \quad a_{21}(c, e) = \frac{\int_{\theta_1 > 2e\theta_2 + c - e} \theta_2 \, d\theta}{\int_{\theta_1 > 2e\theta_2 + c - e} 1 \, d\theta}, \quad a_{22}(c, e) = \frac{\int_{\theta_1 < 2e\theta_2 + c - e} \theta_2 \, d\theta}{\int_{\theta_1 < 2e\theta_2 + c - e} 1 \, d\theta};$$

- third, we express conditions on the parameters $c$ and $e$ so that $\Theta_0(c, e)$ is the bisector of the shifted action profiles $\bm{a}'_i = (a_{11}(c, e) - b, a_{12}(c, e) - b)$ and $\bm{a}'_2 = (a_{21}(c, e) - b, a_{22}(c, e) - b)$; the conditions are:

- the line $\Theta_0(c, e)$ is perpendicular to the segment $[\bm{a}'_1, \bm{a}'_2]$, i.e., the product of the slopes of $\Theta_0(c, e)$ and $[\bm{a}'_1, \bm{a}'_2]$ is $-1$:

$$\frac{a_{22}(c, e) - a_{21}(c, e)}{a_{11}(c, e) - a_{12}(c, e)} = -2e; \quad (4)$$
the middle point of the segment $[a'_1 a'_2]$ is an element of $\Theta_0$, i.e.
\[
\frac{a_{11}(c, e) + a_{12}(c, e) - 2b}{2} = \frac{a_{21}(c, e) + a_{22}(c, e) - 2b}{2} + c - e; \quad (5)
\]

- fourth, solving \([1]\) and \([5]\) given conditions on $c$ and $e$ determined by the case in study ((A), (B), (C) or (D)), we obtain parameterizations $(c, e, b)$ of the possible equilibria. Note that by symmetry, we may restrict our solutions to the cases $b \geq 0$.

**Case (A)**

In case (A), we have
\[
0 \leq c + e \leq 1, \quad 0 \leq c - e \leq 1 \quad \text{and} \quad (c, e) \not\in \{(0, 0), (1, 0)\}. \quad (6)
\]

We compute
\[
a_{11}(c, e) = \frac{3c^2 + e^2 - 3}{6(c - 1)}, \quad a_{12}(c, e) = \frac{3c^2 + e^2}{6c},
\]
\[
a_{21}(c, e) = \frac{3c + e - 3}{6(c - 1)}, \quad a_{22}(c, e) = \frac{3c + e}{6c}.
\]

Condition \([4]\) writes $\frac{c}{c^2 + 3c^2 - 3c} = -2e$. From \([6]\) we obtain either $e = 0$ or
\[
e^2 = -\frac{1}{2} - 3c^2 + 3c \quad (7)
\]
(see Figure 10 below for the conjunction with \([6]\).

![Diagram](image)

**Fig. 10: Condition [4] in Case (A)**

Condition \([5]\) gives then,

- if $e = 0$, then $b = \frac{1-2c}{4}$; this corresponds to decision-making rule considering only the $\theta_1$ dimension (Figure 11 gives an illustration); from \([6]\), it is necessary that $b \in (-\frac{1}{4}, \frac{1}{4})$;
Fig. 11: Decision-making rules based on the first dimension only, with $0 \leq b < \frac{1}{4}$

- if $(c, e) = \left( \frac{1}{2}, \frac{1}{2} \right)$, no condition on $b$; this corresponds to the comparative symmetrical decision-making rule (see Figure 12):

Fig. 12: Comparative decision-making rule, independent of $b \geq 0$

- if $(c, e) \neq \left( \frac{1}{2}, \pm \frac{1}{2} \right)$, then $b = \frac{(2c-1)(-3c+3c^2+e^2)}{(12c-1)c(2c-1)}$; Figure 13 below illustrates the maps $c \mapsto b$, conditional on the value of $e = \pm \sqrt{-\frac{1}{2} - 3c^2 + 3c}$ given by (7); in particular we have an equilibrium at each $b \in [-\frac{2}{9}; -\frac{3}{27}] \cup \left[ \frac{2}{27}; \frac{2}{9} \right]$, that satisfies
  - if $b \in \left[ -\frac{2}{9}, -\frac{\sqrt{3}}{12} \right]$, $c \in \left[ \frac{3}{4}, \frac{3+\sqrt{3}}{6} \right]$, $e \geq 0$,
  - if $b \in \left[ -\frac{\sqrt{3}}{12}, \frac{2}{27} \right]$, $c \in \left[ \frac{3}{4}, \frac{3+\sqrt{3}}{6} \right]$, $e \leq 0$,
  - if $b \in \left[ \frac{2}{27}, \frac{\sqrt{3}}{12} \right]$, $c \in \left[ \frac{3-\sqrt{3}}{6}, \frac{1}{4} \right]$, $e \leq 0$. 

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if \( b \in \left[ \frac{\sqrt{3}}{12}, \frac{2}{9} \right], c \in \left[ \frac{3-\sqrt{3}}{6}, \frac{1}{4} \right], \) \( e \geq 0; \)

The corresponding decision-making rules are illustrated in Figure 5.

Case \( e \geq 0 \)

In case (B) we have

\[
c - e \leq 0, \quad 0 \leq c + e \leq 1 \text{ and } (c, e) \neq (0, 0). \quad (8)
\]

We compute

\[
a_{11}(c, e) = \frac{6e - (c + e)^3}{3(4e - (c + e)^2)}, \quad a_{12}(c, e) = \frac{c + e}{3},
\]
\[
a_{21}(c, e) = \frac{(c - e)^3 - 4e^2(e - 3(1 - c))}{6e(4e - (c + e)^2)}, \quad a_{22}(c, e) = \frac{5e - c}{6e}.
\]

Condition (1) gives \( \frac{2e - e}{e(2e + 2e - 3)} = -2e \), that is \( e = \frac{1}{2} \) or \( c = \frac{e(1-e)}{e+\frac{1}{2}} \) (see Figure 14 below).

Fig. 13: Case (A) with \((c, e) \neq \left( \frac{1}{2}, \pm \frac{1}{2} \right)\)

Case (B)

In case (B) we have

\[
c - e \leq 0, \quad 0 \leq c + e \leq 1 \text{ and } (c, e) \neq (0, 0). \quad (8)
\]

We compute

\[
a_{11}(c, e) = \frac{6e - (c + e)^3}{3(4e - (c + e)^2)}, \quad a_{12}(c, e) = \frac{c + e}{3},
\]
\[
a_{21}(c, e) = \frac{(c - e)^3 - 4e^2(e - 3(1 - c))}{6e(4e - (c + e)^2)}, \quad a_{22}(c, e) = \frac{5e - c}{6e}.
\]

Condition (1) gives \( \frac{2e - e}{e(2e + 2e - 3)} = -2e \), that is \( e = \frac{1}{2} \) or \( c = \frac{e(1-e)}{e+\frac{1}{2}} \) (see Figure 14 below).

Fig. 14: Condition (1) in Case (B)
Condition (5) gives then

- $e = \frac{1}{2}$, $c = \frac{1}{2}$ and no condition on $b$; this corresponds to the comparative symmetrical equilibrium (Figure 12);

- $b = -\frac{16e^4 + e^2 + 1}{(2e-1)(2e+1)(16e^2 + 7e + 3)}$; Figure 15 below represents the map $e \mapsto b$; we have an equilibrium at each $b \in (-\infty; -\frac{2}{9}] \cup \left[\frac{2}{9}; +\infty\right)$ with $c = \frac{e(1-e)}{e^2 + 1}$ that satisfies
  
  - for each $e \in \left[\frac{1}{4}; \frac{1}{2}\right)$, a positive bias $b$ that range from $\frac{2}{9}$ to $+\infty$ as $e$ ranges from $\frac{1}{4}$ to $\frac{1}{2}$,
  
  - for each $e \in \left(\frac{1}{2}; 1\right]$, a negative bias $b$ that ranges from $-\infty$ to $-\frac{2}{9}$ as $e$ ranges from $\frac{1}{2}$ to 1.

The corresponding decision-making rules are illustrated in Figure 5.

\[ b \]

\[ \frac{2}{9} \quad \frac{1}{2} \quad 1 \]

\[ e \]

Fig. 15: The map $e \mapsto b$ in Case (B)

Case (C)

In case (C), we have

\[ c + e \leq 0, \quad 0 \leq c - e \leq 1 \text{ and } (c, e) \notin \left\{ (0, 0), \left(\frac{-1}{2}, \frac{-1}{2}\right) \right\}. \tag{9} \]

Also, by relabeling $\theta_1$ and $\theta_2$ we may w.l.o.g. restrict $e$ to $[-\frac{1}{2}; 0]$ (see Figure 9: the case (C) with $e \in [-\frac{1}{2}; 0]$ is symmetrical to Case (C) with $e \in [\frac{-1}{2}; 0]$). We compute

\[ a_{11}(c, e) = -\frac{c^3 - 6e + 3c^2e - 3ce^2 + e^3}{3(e^2 - 2ce + 4e + c^2)}, \quad a_{12}(c, e) = \frac{c-e}{3}, \]

\[ a_{21}(c, e) = \frac{-c^3 + 3c^2e + 12e^2 - 3ce^2 + e^3}{6e(e^2 - 2ce + 4e + c^2)}, \quad a_{22}(c, e) = \frac{e-c}{6e}. \]

Condition (4) gives $\frac{c+2e}{e(2e-2e+3)} = -2e$ so that either $e = \frac{-1}{2}$, or $c = \frac{2e+2e^2}{2e-1}$. Figure 16 below illustrates these condition under (9).
Condition (5) gives then

- if $e = -\frac{1}{2}$, from (9), then $c$ ranges from $-\frac{1}{2}$ to $\frac{1}{2}$ with a corresponding bias $b = \frac{9 - 26c + 12c^2 + 8c^3}{84 - 48c^2 - 48c}$ ranging from $\frac{1}{4}$ to $0$; this corresponds to a (asymmetrical with bias) aggregate equilibrium;

- for each $e \in (-\frac{1}{2}; -\frac{1}{4}]$ (in Figure 15, the cases $e \in [-1; -\frac{1}{2})$ are symmetrical with respect to the relabeling of $\theta_1$ and $\theta_2$), and corresponding $c = \frac{2e + 2c^2}{2e - 1}$, an equilibrium with bias $b = \frac{1 + c^2 + 16c^4}{(2e - 1)^2(16e^2 - 7e + 4)}$. Here again, the corresponding decision-making rules are illustrated in Figure 15.

Fig. 16: Condition (4) in Case (C)

Fig. 17: Deciding between $\theta_1 + \theta_2 \geq c$ or not, with $0 \leq b < \frac{1}{4}$
Case (D)

Case (D) is derived from Case (C). Indeed, since in case (C), an equilibrium exists only with positive bias, we have a symmetrical equilibrium in case (D) with an opposite bias, hence a negative bias.

Proof of Lemma 1

Let $D \in \varepsilon(b)$ for some $b \in \mathbb{R}^2$, and let $a_i, i = 1, 2$ be the corresponding actions. Let $b' \in \mathbb{R}^2$. From $\|x\|^2 = x \cdot x$, we obtain

$$-\|a_1 - (\theta + b')\|^2 \geq -\|a_2 - (\theta + b')\|^2$$

$$\iff -\|(a_1 - b - \theta) - (b' - b)\|^2 \geq -\|(a_2 - b - \theta) - (b' - b)\|^2$$

$$\iff -\|a_1 - (\theta + b)\|^2 \geq -\|a_2 - (\theta + b)\|^2 - 2(b' - b) \cdot (a_1 - a_2).$$

Therefore a decision-making rule associated with a game $\Gamma_b$ discloses upon the same states than $D$, i.e. is equal to $D$, if and only if $(b' - b) \cdot (a_1 - a_2) = 0.$

Proof of Proposition 4

We first show that for any conflict of interest, there exists an influential equilibrium.

We will give a geometrical proof. In Figure 19 for each decision-making rule $D^{as}(b)$, we have represented the orthogonal projection $\pi(a_1 - b)a_1 \rightarrow$ of the vector $(a_1 - b)a_1$ on the line $(a_1a_2)$. The bold graph of Figure 19 represents the sets of $\pi(a_1 - b)$ for $b \in [\frac{9}{184}, \frac{\sqrt{3}}{12}]$. From Lemma 1 for each $b \in [\frac{9}{184}, \frac{\sqrt{3}}{12}]$, the perpendicular to $(a_1a_2)$ passing through $\pi(a_1 - b)$ (represented as the dashed line in Figure 19) represents the sets $-B(D^{as}(b))$. 
Therefore, the grayed region of Figure 20 represents all bias $-b$ for which there exists $b \in [\frac{9}{184}, \frac{\sqrt{3}}{12}]$ such that $D^{as}(b)$ is a decision-making rule that occurs in an equilibrium of $\Gamma_b$. Similarly, Figure 21 represents all bias $-b$ for which there exists $b \geq \frac{\sqrt{3}}{12}$ such that $D^{as}(b)$ is a decision-making rule that occurs in an equilibrium of $\Gamma_b$. 

Fig. 20: Senders’ bias $b$ compatible with $D^{as}(b)$, $\frac{9}{184} \leq b < \frac{\sqrt{3}}{12}$
Fig. 21: Senders’ bias $b$ compatible $D^{as}(b)$, $b > \frac{\sqrt{3}}{12}$

According to Lemma 2, a quarter-turn rotation (in the trigonometric sense) of the set of bias represented in Figure 20 represents the set of bias $-b$ for which there exists $b \in \left[\frac{9}{184}, \frac{\sqrt{3}}{12}\right]$ such that $\rho(D^{as}(b))$ is a decision-making rule that occurs in an equilibrium of $\Gamma_b$. This applies to each rotations $\rho^k(D^{as}(b))$. In Figure 22 all such bias $-b$ for which there exists $b \geq \frac{9}{184}$ such that there exists $k \in \{0, 1, 2, 3\}$ such that $\rho^k(D^{as}(b))$ is a decision-making rule that occurs in an equilibrium of $\Gamma_b$.

Fig. 22: Senders’ bias $b$ compatible with one of the $\rho^k(D^{as}(b))$ for $k \in \{0, 1, 2, 3\}$

Figure 22 shows that for any sufficiently large bias ($\|b\| \geq \sqrt{\frac{18}{184}}$ suffices), there exists at least two of the decision-making rules $\rho^k(D^{as}(b))$ that occurs in equilibrium of $\Gamma_b$.  

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Similarly, according to Lemma 1, the symmetrical decision-making rules and their deviations \( C(c), A(c) \) and \( H_i(c), i \in \{1, 2\} \) occurs as equilibrium of \( \Gamma_b \) for \(-b\) represented in grayed regions of Figure 23.

Fig. 23: Senders’ bias \( b \) compatible with one of the \( C(c), A(c) \) and \( H_i(c), i \in \{1, 2\} \)

Figures and 23 show that for any bias \( b \in \mathbb{R}^2 \), there exists an influential decision-making rule \( D \) such that \( D \) occurs in equilibrium of \( \Gamma_b \).

Next we show that any influential decision making rule is an influential decision making rule associated with a symmetrical bias or a bias with opposed components.

Let \( D \) be a decision-making rule in equilibrium of a game \( \Gamma_b \) with \( b = (b_1, b_2) \in \mathbb{R}^2 \). Let \( a_1, a_2 \) be the corresponding induced actions of the Receiver, and set \( \pi_D \) the orthogonal projection on the line \( (a_1, a_2) \) of induced actions.

Then \( D \) occurs as a decision-making rule of \( \Gamma_{b'} \) if and only if \( \pi_D(b') = \pi_D(b) \).

Let \( \tilde{b} \neq 0 \) be any vector normal to \( (a_1, a_2) \), and consider, for any \( \lambda \in \mathbb{R} \),

\[
b(\lambda) = \pi_D(b) + \lambda \tilde{b}.
\]

For any \( \lambda \in \mathbb{R} \), we have \( \pi_D(b(\lambda)) = \pi_D(b) \) so that \( D \) occurs as a decision-making rule of \( \Gamma_{b(\lambda)} \).

Thus we have to show that there exist \( \lambda \in \mathbb{R} \), such that \( b(\lambda) \) is of the form \( (b, b) \), or there exist \( \lambda \in \mathbb{R} \) such that \( b(\lambda) \) is of the form \( (b, -b) \).

Write \( b(\lambda) = (b_1(\lambda), b_2(\lambda)) \in \mathbb{R}^2 \), \( \pi_D(b) = (b_1^*, b_2^*) \in \mathbb{R}^2 \) and \( \tilde{b} = (\tilde{b}_1, \tilde{b}_2) \in \mathbb{R}^2 \).
Since $\tilde{b}$ is normal to $(a_1a_2)$, we have

$$b_1^\pi \tilde{b}_1 + b_2^\pi \tilde{b}_2 = 0.$$ 

If $\tilde{b}_1 = \tilde{b}_2$, then $(a_1a_2)$ is normal to the first diagonal $\theta_1 = \theta_2$, and therefore $D$ is a deviation of the symmetrical comparative decision making rule $C$. If $\tilde{b}_1 \neq \tilde{b}_2$, then setting $\lambda = \frac{b_1^\pi - b_2^\pi}{b_2 - b_1}$, we obtain

$$b_1(\lambda) = b_1^\pi + \frac{b_1^\pi - b_2^\pi}{b_2 - b_1} \tilde{b}_1 = \frac{b_1^\pi \tilde{b}_2 - b_2^\pi \tilde{b}_1}{b_2 - b_1} = b_2^\pi + \frac{b_1^\pi - b_2^\pi}{b_2 - b_1} \tilde{b}_2 = b_2(\lambda).$$

Hence $b(\lambda)$ is of the form $(b, b)$.