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Competing Ecosystems and Quality Investment

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Competing ecosystems and quality investment

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Abstract

Traditional firms competing in a primary market may expand into a secondary market that generates user data and enhances the quality of the primary product. This paper examines how competition between such rival ecosystems affects market outcomes and welfare. Using a Hotelling framework with two symmetric ecosystems that each offer a primary product and a secondary data-rich product, I show that the size of the secondary market is key. When the secondary market is small, ecosystems invest less in quality than in a benchmark with only a primary market and earn higher profits at the expense of consumers. As the secondary market grows, quality investment rises and the welfare ranking can reverse. I further show that expansion into a secondary market need not create a trade-off between profits and consumer surplus: when the ecosystems' secondary products are sufficiently differentiated, both profits and consumer surplus can exceed their benchmark levels. These findings inform policy debates on digital adoption, market structure, and ecosystem regulation.

Keywords: Competing ecosystems ; quality investment ; data-driven quality.
JEL Classification: L13, L51, D43, O31, Q16.

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1 Introduction

Firms traditionally supplying a primary market increasingly expand into a secondary digital market that enables to gather usage data related to the primary products, thereby operating as *ecosystems*. In such ecosystems, product quality draws on two interrelated sources: ex ante investment and ex post improvements driven by user-generated data. This creates a cross-market dynamic. Higher investment raises quality and demand in the primary market, while adoption in the digital market generates data that further raises primary-product quality and, in turn, demand. The OECD identifies this data-driven innovation as a new source of growth, with data and analytics serving as inputs into innovation in ways comparable to R&D (OECD, 2015). Does the prospect of data-driven quality raise or dampen ecosystems' incentives to invest in R&D-driven quality, and what does the answer imply for profits and consumer welfare?

Much of the literature on ecosystems and data-related markets studies how data can generate asymmetries that firms may leverage to extend market power across markets, often through data-driven network effects or tipping mechanisms (Condorelli and Padilla, 2024; Choi *et al.*, 2026; De Cornière and Taylor, 2024; Krämer and Shekhar, 2025). Closer to our setting where firms compete in both price and quality, but still featuring asymmetric firms, Rhodes *et al.* (2026) examine how data advantages affect quality investment incentives when a generalist ecosystem competes in many markets with specialist firms. Such asymmetries are useful for understanding the strategies of big tech companies such as Google or Apple. However, they are less representative of industries in which competition takes place among a small number of established firms with comparable positions in the primary market and that are increasingly expanding into nascent digital markets, where adoption remains limited and big-tech platforms are absent.

This paper studies competition between such newly formed ecosystems. I consider a setting in which competing firms are symmetric and face similar opportunities to collect and exploit user data through a secondary digital market. The analysis focuses on how competition shapes the trade-off between ex ante R&D investment and ex post data-driven quality improvements. I show that the size and competitiveness of the digital market play a central role in determining firms' incentives to invest, with important consequences for market outcomes, profits, and consumer welfare.

Agricultural input markets offer a leading illustration. Firms traditionally specialized in seeds and chemical inputs are expanding into digital advisory services, known as Decision Support Systems (DSS), such as Bayer's Climate FieldView and BASF's xarvio Field Manager (Sauvagerd *et al.*, 2024). These technologies provide agronomic recommendations while generating detailed farm-level data that can be used to refine their products (Carbonell, 2016). Climate FieldView's terms of use state that "I also may use your customer farm data for research and development purposes, such as to improve Climate and Bayer's agronomic or scientific knowledge, [and] develop and improve our products and services." Innovation in this sector therefore increasingly relies on two interrelated sources: ex ante investment in input quality and ex post data-driven improvement derived from the use of digital tools.

The sector also has the oligopolistic structure that our framework assumes: a small number of large firms compete in the primary market while simultaneously developing their own digital services (OECD, 2018; Deconinck, 2019; Sauvagerd *et al.*, 2024). Because these firms have comparable opportunities to collect and exploit data, the setting is precisely one of rivalry between symmetric ecosystems rather than dominance by a single one.

Another noteworthy feature of this sector is the limited scope for bundling input sales with agronomic advisory services, owing to regulatory and organizational constraints. In France—the EU's leading agricultural producer[1]—the EGAlim framework prohibits such ecosystems from jointly providing phytosanitary advice and products, while distributors may do so only under strict certification and organizational-separation conditions. Comparable separation principles apply in other major producing countries, including Germany and Denmark.[2] I therefore abstract from input–advisory bundling, which further distinguishes our analysis from the literature, where tying and bundling can be central channels through which firms extend market power (Choi *et al.*, 2026).

The questions extend beyond agriculture to other oligopolistic industries. Healthcare offers a parallel example: several major pharmaceutical firms are moving beyond drug production into digital remote-care and clinical decision-support platforms whose use generates patient data that feeds back into treatment and product development. Roche's Navify Tumor Board lets hospitals coordinate multidisciplinary cancer-treatment decisions on shared clinical data and analytics, while Sanofi's diabetes tools such as MyStar Connect support remote glucose monitoring and insulin adjustment between patients and clinicians. AstraZeneca, in turn, develops digital health ecosystems through Evinova for clinical-trial support, remote patient monitoring, and related digital services. In each case, adoption of the digital service generates usage data layered above a traditional pharmaceutical market—mirroring the agricultural pattern in which digital advisory tools both serve users directly and can refine the primary products.[3]

Motivated by these examples, I develop a stylized model of competition between two ecosystems operating in both a primary market and a secondary digital market. The framework builds on a Hotelling model with two independent consumer markets, allowing

[1] France accounts for ∼18% of EU agricultural output and ranks first in wheat production (Eurostat, 2024; USDA FAS, 2025).

[2] France: Décret n°2020–1265 du 16 octobre 2020 (legifrance); Germany: DüngG §12; Denmark: Miljøstyrelsen IPM guidelines (2023).

[3] Roche NAVIFY Tumor Board: Roche launch release and Roche digital health solutions; Sanofi MyStar Connect: Sanofi announcement and Sanofi diabetes digital care; AstraZeneca Evinova: Evinova and AstraZeneca launch release.

preferences for primary products and digital products to differ. Consistent with evidence on the diffusion of digital farming technologies, only a fraction of farmers purchasing inputs also adopt digital products (Kroupová *et al.*, 2025).

Ecosystems first choose product quality in the primary market. Adoption in the digital market then generates ecosystem-specific data that translates into an improvement in product quality in the primary market. This data-driven component of quality is realized after market interaction and depends on equilibrium demand for digital services. A key determinant of this mechanism is the size of the digital market: since only a fraction of farmers adopt digital products, the magnitude of data-driven improvement is limited and exogenous.

The model delivers four sets of results, all organized around the size of the digital market.

First, firms always subsidize digital adoption to collect data, and this creates two opposing forces on quality investment. When a firm raises its quality it can charge more for its primary product and subsidize digital users more aggressively, since the data those users supply now raise its primary-market share by more. The rival, less able to subsidize, must instead cut its primary-market price more sharply to defend its share. The sharper primary-market price response depresses investment; the softer digital-market subsidy response raises it. The size of the data market decides which force dominates: when the market is small, ecosystems invest less than in a benchmark without digital services; once it is large enough, investment rises above the benchmark.

Second, the same forces make equilibrium profit U-shaped in the size of the digital market. Profit can be written as benchmark primary-market profit, minus the losses firms incur subsidizing digital adoption, minus quality-investment cost. The subsidy losses are themselves hump-shaped in digital-market size: they grow as the market expands from zero, because firms compete fiercely for the few data-supplying consumers and drive secondary prices negative, but they shrink again once the market is large enough that digital prices recover. Investment cost, by contrast, rises throughout. Netting these out, profit falls as the data market first grows and then rises once it is large: a U-shape. In the baseline, profit sits above the no-digital market benchmark only while the market is small; it then decreases below and then increases again but without returning above the benchmark. Consumer surplus instead rises with digital-market size and is above the benchmark once the market is large. Digitalization therefore need not benefit ecosystems, and benefits consumers only once the digital market is sufficiently developed.

The profit–consumer-surplus trade-off need not persist, and the U-shape property of profits is key. When the secondary products are sufficiently differentiated, rivalry for data softens: firms incur smaller digital losses and invest less excessively, which lifts the entire profit curve—and in particular its rising branch—upward. For a large enough data market, that rising branch can be above the benchmark while quality stays high enough to keep consumers above theirs. There is then a region in which profits and consumer surplus exceed their no-digital-market levels at once, so the emergence of competing ecosystems makes firms and consumers better off together. These mechanisms survive partial R&D spillovers into digital quality and a weaker data-driven quality channel.

Finally, I let firms choose whether to expand on the digital market for a fixed cost. Expansion is a dominant strategy when this cost is low, so competing ecosystems emerge endogenously; for intermediate costs only one firm expands and becomes a digital incumbent. Mutual expansion is a prisoner's dilemma exactly when profits sit below the benchmark—each firm expands to secure a data advantage, yet both would be better off abstaining. Sufficient product differentiation can reduce the dilemma, but does not resolve it completely.

These mechanisms speak directly to current policy debates on the digital transition of agriculture, and our analysis points to several levers. First, policies that expand the digital market—through training, connectivity, and adoption support, as promoted by the European Commission's Common Agricultural Policy—allow data-driven and R&D complementarities to materialize and raise consumer welfare once the digital market size is sufficiently great (European Commission Joint Research Centre, 2025). Second, policies that affect the fixed cost of creating digital products—through regulation and interoperability obligations, as in the EU Data Act—shape entry incentives and can prevent welfare losses by moderating ecosystem formation at early stages of digital-market development (European Parliament and Council of the European Union, 2023). Third, the degree of competition between ecosystems matters: greater product differentiation—for example, decision-support tools specialized by crop—softens the rivalry for data while preserving incentives to invest, and can make both ecosystems and consumers better off.

To a lesser extent, these insights resonate with recent discussions in *The Future of European Competitiveness* report led by Mario Draghi, which stresses that merger policy should ensure that acquisitions do not dampen innovation incentives and argues that merging firms should be required to demonstrate that a transaction will foster, rather than hinder, innovation (Draghi, 2024). In digital agriculture, ecosystems are often built through the acquisition of specialized firms to expand into the digital market—as with Bayer-Monsanto's acquisition of The Climate Corporation, Syngenta's purchase of FarmShots, or John Deere's acquisition of Blue River Technology.[4] Such acquisitions can be represented by the fixed entry cost in our model, which therefore provides a framework to assess when ecosystem formation through acquisition is likely to enhance or dampen welfare.

[4]Acquisition sources: Monsanto acquires The Climate Corporation (2013): https://www.farm-equipment.com/articles/9462-monsanto-to-acquire-the-climate-corporation; Syngenta acquires FarmShots (2018): https://globalaginvesting.com/syngenta-acquires-farmshots/; John Deere acquires Blue River Technology (2017): https://www.cnbc.com/2017/09/06/deere-is-acquiring-blue-river-technology-for-305-million.html

The remainder of the paper is organized as follows. Section 2 reviews the related literature. Section 3 presents the model. Section 4 characterizes the benchmark and the equilibrium with competing ecosystems. Section 5 studies how the degree of competition in the digital market affects the results. Section 6 analyzes endogenous expansion. Section 7 discusses the policy implications, Section 8 examines robustness to alternative specifications, and Section 9 concludes.

2 Related literature

The present paper contributes to several strands of the literature on digital ecosystems and quality investment.

First, a large body of work studies how firms active across multiple markets exploit cross-market linkages to strengthen their competitive position. These linkages operate through data strategies, as in Rhodes *et al.* (2026), where a multiproduct ecosystem uses data from one product to raise the quality of its others against single-product rivals, and as in Bhargava *et al.* (2026), who show that a specialist firm may willingly share its data with a generalist entrant; through vertical integration and foreclosure between competing ecosystems (Bisceglia *et al.*, 2022); and through tying (Choi *et al.*, 2026). Related to merger between firms and cross-market mechanism is De Cornière and Taylor (2024), who study mergers between firms in data-connected markets, where data generated in one market improves product quality or extracts surplus in another, and show that the welfare effects hinge on whether data could be traded absent the merger; relatedly, Chen *et al.* (2022) link a data-collection market to a data-application market through a consumption synergy and characterize when the resulting personalization raises or lowers prices. A further strand examines how such linkages drive the strategic formation of ecosystems: a dominant firm may expand a complementary market and price aggressively to acquire data protecting its primary position (Condorelli and Padilla, 2024), demand-side linkages may raise the likelihood of repeat purchases from a multi-product firm (Heidhues *et al.*, 2024), and network centrality shapes ecosystem competition (Jeon *et al.*, 2025). In Krämer and Shekhar (2025) an ecosystem monopolizes a primary market, and uses data from it to improve its product in a secondary market, where it competes 'a la Cournot with a single-product firm. They find that consumers are always better off when the ecosystem is forced to share primary-market data with its rival. In contrast to this literature, our two firms are fully symmetric: each operates in both markets with identical access to data, so the outcomes are driven by symmetric rivalry rather than from a dominant market position or integration. As shown earlier, this framework of rivalry between traditional firms better fits several industries where there are no big tech companies.

The mechanism through which the adoption of a digital service generates firm-specific data that feeds back into the quality, and hence the demand, of the primary product is closely related to the literature on data-driven competition. In this strand of the literature, Prüfer and Schottmüller (2021) and Hagiu and Wright (2023) study dynamic duopoly models in which single-product firms use customer data generated by past transactions to improve their products. In Prüfer and Schottmüller (2021), firms compete in quality and the cost of providing quality decreases with the number of customers served in the previous period. In Hagiu and Wright (2023), firms compete in price and product value increases with the number of past users. Despite these differences, both papers show that data-driven product improvements create self-reinforcing dynamics that can lead to market tipping.

I share with both contributions the premise that usage generates data that can indirectly improve quality, but I depart from them in several aspects. First, symmetric firms in our model act simultaneously rather than dynamically, preventing the emergence of a market leader. Section 6 nevertheless analyzes endogenous expansion and how we can get market tipping by a single firm for some market parameters. Second, quality is a combination of both ex ante R&D investment and ex post data-driven improvement, rather than pure data-driven quality enhancement (Hagiu and Wright, 2023) or indirect data-driven investment efficiency (Prüfer and Schottmüller, 2021). Third, data are generated in a separate segment of exogenous size, which turns out to be the key determinant of whether the ecosystem increases or decreases investment relative to a benchmark without digital market. Finally, in my steting firms compete in both prices and quality choices.

The focus on endogenous quality investment links to Mantovani and Ruiz-Aliseda (2016), who study complementors collaborating to raise system quality and identify a "dark side" in which collaboration lowers joint profitability. In our setting the two firms are rivals rather than complementors, and quality combines an endogenous ex ante investment with an ex post improvement driven by the data that digital adoption generates. The paper is also close to Allain *et al.* (2025), who study investment incentives under symmetric platform competition and show that investment may be stronger under a user-funded than an advertising-funded model. The setting resembles their user-funded case, since consumers pay for access to the primary product, but firms in our setting compete through a single quality choice that feeds a primary market, with no multihoming and no two-sided network effects. I also compare investment with a benchmark where the digital market is absent, therefore distancing our contribution to two-sided markets where the two sides are necessary for platform's existence as an intermediary.

Finally, the equilibrium feature that firms price the digital service below cost to harvest data connects our analysis to the literature on loss leading and below-cost pricing (Chen and Rey, 2012). Whereas in that literature a product is sold below cost to attract cross-shopping consumers or to exploit the uninformed, in our setting the digital service is subsidized to accumulate data that raises the value of the primary product; the "loss leader" is therefore a data-collection device rather than a demand-side bait, and its intensity depends on the size and competitiveness of the digital segment.

3 Model

I consider a two-market Hotelling framework with differentiated products. There are two firms, $i \in \{A, B\}$, each active in both a primary market (market 1) and a secondary market (market 2). In market 1, firms supply primary products A_1 and B_1; in market 2, they supply data-rich products A_2 and B_2, respectively.

Primary market. Market 1 has unit mass of consumers uniformly distributed on $[0, 1]$. Firms supply horizontally differentiated primary products with intrinsic value $v_1 \geq 0$. Firm A is located at 0, and firm B is located at 1. Firms set prices p_{A_1} and p_{B_1}, respectively, and can also invest in product quality to enhance product value. Let $q_i \geq 0$ denote the quality chosen by firm i. Investment entails a convex cost:

$$C(q_i) = \frac{q_i^2}{2\eta}, \tag{1}$$

where $1 \leq \eta \leq 7$ denotes investment efficiency: a higher η lowers the marginal cost of quality and therefore strengthens the incentive to invest. This quadratic specification is standard in models of quality competition and digital ecosystems (Rhodes *et al.*, 2026). Throughout the main analysis of Sections 4 and 5 we maintain $\eta \leq 7$, which ensures a concave quality-stage objective.[5] However, the endogenous-expansion analysis of Section 6 requires the tighter condition $\eta \leq 3$ for an interior primary market split, and the R&D-spillover extension of Section 8 needs also a tighter condition that depends on the new parameter.

I assume that v_1 is sufficiently large to ensure full market coverage in equilibrium. Aside pure R&D investment on quality, each firm benefits from data-driven quality investment from the data accumulated in the secondary market. For simplification, I suppose the quality of their respective products increases of the amount of their respective demand on the secondary market. I denote them respectively d_A and d_B. Section 8 examines lower levels of data-driven quality enhancement, capturing potential frictions in the data accumulation process or discounting of the additional later value brought by data-driven quality.

[5]This is the binding restriction: the benchmark requires only $\eta < 9$, and the transport-cost variant $\eta < 9 - 2/t$, which exceeds 7 for every $t > 1$, so the closed range $\eta \in [1, 7]$ of Proposition 4 is admissible there.

A consumer located at $x \in [0,1]$ derives utility:

$$\begin{aligned} U_{A_1}(x) &= v_1 + q_A + d_A - p_{A_1} - x, && \text{if buying from A,} \\ U_{B_1}(x) &= v_1 + q_B + d_B - p_{B_1} - (1-x), && \text{if buying from B.} \end{aligned} \tag{2}$$

Secondary market. Market 2 has mass $\rho \in (0,1]$, capturing that only a fraction of consumers in the first segment also purchase the second product. For instance, only some consumers may possess sufficient digital literacy to derive value from such services, or access to these tools may be restricted to specific distribution channels (e.g., certain cooperatives), thereby limiting the effective market size (Kroupová *et al.*, 2025). Consumers are uniformly distributed on a segment $[0, \rho]$, independently of their location in market 1. This assumption is standard in the literature of data-related markets (Cong and Matsushima, 2026).

Firms supply differentiated products A_2 and B_2 with intrinsic value $v_2 \geq 0$, with firm A (respectively B) located at 0 (respectively ρ). I assume that v_2 is sufficiently large to ensure full market coverage in equilibrium. Firms set prices p_{A_2} and p_{B_2}, respectively. For simplification, product quality in the secondary market is not affected by the firm's primary quality. This helps us to more neatly characterize the main effects at stake. Section 8 relaxes this assumption by allowing for imperfect quality spillovers across markets, and shows that our main results remain. For example, R&D investment is needed to understand the agronomic properties to create primary products like seeds or pesticides which also indirectly serves the algorithm of the decision support system.

Consumers have unit demand in each market and make independent purchasing decisions across markets.[6] A consumer located at $y \in [0, \rho]$ obtains utility:

$$\begin{aligned} U_{A_2}(y) &= v_2 - p_{A_2} - y, && \text{if buying from A,} \\ U_{B_2}(y) &= v_2 - p_{B_2} - (\rho - y), && \text{if buying from B.} \end{aligned} \tag{3}$$

To summarize, the secondary data-rich product generates two sources of value. First, it provides a direct benefit to consumers who purchase it, in addition to the utility they derive from the primary product. Second, it indirectly increases the valuation of the primary product through data-enhanced quality improvements. For instance, a farmer purchasing a decision-support tool may receive better agronomic advice, leading to higher crop yields, while also benefiting from improved seeds, fertilizers, or pesticides whose quality has been enhanced through data collection. Importantly, farmers who purchase only the primary products benefit from the data-driven quality improvements generated

[6] Note that our assumptions could also model a setting where consumers in the secondary market differ from those in the primary market. For example, doctors may use digital diagnostic tools whose collected data improve the quality of pharmaceutical products consumed by patients. Patients buy the primary products while doctors buy the data-rich products.

through data collection, but do not directly benefit from using the decision-support tools.

Timing. The game unfolds in three stages as follows:

1. Firms simultaneously choose quality levels q_A and q_B.
2. Firms simultaneously set prices $p_{A_1}, p_{B_1}, p_{A_2}, p_{B_2}$.
3. Consumers make purchase decisions in each market.

4 Analysis and Results

4.1 Benchmark case without secondary market

I first derive a benchmark in which firms operate exclusively in the primary market, so that product quality is determined solely by ex ante investment. I solve the game by backward induction. Consumer demand follows the standard Hotelling condition, $q_A - p_{A_1} - x = q_B - p_{B_1} - (1-x)$, yielding a marginal consumer location $\tilde{x}^b(p_{A_1}, p_{B_1}) = \frac{1}{2} + \frac{1}{2}(q_A - q_B + p_{B_1} - p_{A_1})$. I further denote by Π_i operating profits before investment costs and by π_i net profits after investment costs. At the pricing stage, investment cost is sunk and firm operating profits are $\Pi_A = p_{A_1}\tilde{x}^b$ and $\Pi_B = p_{B_1}(1-\tilde{x}^b)$, from which standard best-response pricing gives $p_A^b = 1 + \frac{1}{3}(q_A - q_B)$ and $p_B^b = 1 + \frac{1}{3}(q_B - q_A)$. Substituting these equilibrium prices into the profit functions and incorporating the sunk investment costs yields sub-game equilibrium profits $\pi_A(q_A, q_B) = \frac{(q_A - q_B + 3)^2}{18} - \frac{q_A^2}{2\eta}$ and $\pi_B(q_A, q_B) = \frac{(q_B - q_A + 3)^2}{18} - \frac{q_B^2}{2\eta}$, and solving for quality investments gives a symmetric outcome $q_A^b = q_B^b = \eta/3$, provided the cost efficiency is not too large $\eta \leq 9$. At this benchmark, equilibrium prices and quantities are symmetric, $p_A^b = p_B^b = 1$ and $\tilde{x}^b = 1/2$, with corresponding firm profits $\pi_A^b = \pi_B^b = (9-\eta)/18$ and consumer surplus $CS^b = v_1 + \eta/3 - 5/4$. This setting provides a baseline for comparison with ecosystems framework where data-driven quality may influence pricing or quality investment decisions.

4.2 Main model with secondary market

I now consider the regime in which both firms operate in the secondary (digital) market, forming competing ecosystems. In this case, product quality in the primary market depends both on ex ante investment and on data generated through digital adoption.

In the secondary market, the indifferent consumer satisfies

$$v_2 - p_{A_2} - y = v_2 - p_{B_2} - (\rho - y),$$

which implies

$$\tilde{y}(p_{A_2}, p_{B_2}) = \frac{\rho}{2} + \frac{1}{2}(p_{B_2} - p_{A_2}).$$

In the primary market, the indifferent consumer now satisfies

$$q_A + d_A - p_{A_1} - x = q_B + d_B - p_{B_1} - (1 - x),$$

I suppose consumers have rational expectations and anticipate that data generated in the secondary market equals realized demand:

$$d_A = \tilde{y}, \qquad d_B = \rho - \tilde{y},$$

which yields

$$\tilde{x}(p_{A_1}, p_{A_2}, p_{B_1}, p_{B_2}) = \tilde{x}^b(p_{A_1}, p_{B_1}) + \left(\tilde{y}(p_{A_2}, p_{B_2}) - \frac{\rho}{2}\right).$$

Thus, primary-market demand depends on relative data accumulation. A firm attracting more users in the digital market enjoys a higher market share in the primary market with unit rate $\partial\tilde{x}/\partial\tilde{y} = 1$, meaning one more consumer on the digital market attracts one more consumer on the primary market via data-driven quality effect.

At this stage the cost of investment is sunk and I can omit it, operating profit of firm A is $\Pi_A = p_{A_1}\tilde{x} + p_{A_2}\tilde{y}$ and the first-order conditions for firm A are:

$$\frac{\partial \Pi_A}{\partial p_{A_1}} = \tilde{x}(p_{A_1}, p_{B_1}) + p_{A_1}\frac{\partial \tilde{x}}{\partial p_{A_1}} = 0, \tag{4}$$

$$\frac{\partial \Pi_A}{\partial p_{A_2}} = \tilde{y}(p_{A_2}, p_{B_2}) + \left(p_{A_2} + \underbrace{p_{A_1}\frac{\partial \tilde{x}}{\partial \tilde{y}}}_{\text{data-driven effect}}\right)\frac{\partial \tilde{y}}{\partial p_{A_2}} = 0. \tag{5}$$

Equation (4) has the structure of a standard Hotelling pricing condition. Equation (5) makes the ecosystem price interaction across markets more explicit. When firm A lowers p_{A_2}, it attracts additional consumers in the secondary market (usual price effect), each contributing revenue p_{A_2}. At the same time, because digital adoption feeds back into perceived quality in the primary market, the firm also gains consumers in the primary market (indirect data-driven quality effect) at rate $\partial\tilde{x}/\partial\tilde{y}$. The value of these additional primary-market consumers is p_{A_1}, which explains why this term expands the pricing condition for the digital product.

Although the overall term $p_{A_2} + p_{A_1}\frac{\partial \tilde{x}}{\partial \tilde{y}}$ must be positive, this does not imply that the optimal price in the secondary market must be positive. On the contrary, the firm may optimally set a negative price on the secondary market, anticipating that the additional consumers attracted in the primary market generate sufficient profit to compensate for losses incurred in the digital market.

Solving the pricing stage yields the equilibrium prices:

$$p_{A_1} = 1 + \frac{3}{7}(q_A - q_B), \qquad p_{B_1} = 1 - \frac{3}{7}(q_A - q_B), \tag{6}$$

$$p_{A_2} = \rho - 1 - \frac{1}{7}(q_A - q_B), \qquad p_{B_2} = \rho - 1 + \frac{1}{7}(q_A - q_B). \tag{7}$$

An immediate observation is that prices in the secondary market can indeed be below marginal cost. This reflects that firms do not value digital participation primarily for its direct revenue but for the data-driven quality effect it generates: attracting consumers in the secondary market increases demand in the primary market. Consequently, firms are willing to subsidize adoption in the digital market to accumulate data. This below-cost pricing can be viewed as an analog of classic loss leading (Chen and Rey, 2012): however, in our setting the digital service is subsidized not to attract cross-shoppers but to accumulate data that raises primary-market value.

Another noteworthy observation is that prices respond to quality differences in opposite ways across the two markets, and that primary-market prices are more sensitive than in the benchmark. In the benchmark, quality affects only primary-market demand. In the ecosystem, quality also raises the value of attracting users in the secondary market, since their data feed back into primary-market demand; a marginal change in quality therefore shifts competitive incentives in both markets at once. Consider firm i gaining a quality advantage. In the secondary market it subsidizes adoption more aggressively— its secondary price falls—because each additional digital user is now worth more: the data it supplies, combined with i's higher quality, translate into more profitable primary-market sales. In the primary market, this reinforced lead lets i raise its price more steeply than its intrinsic quality alone would warrant. The rival faces the reverse. On the primary market it cuts its price more aggressively than in the benchmark, since the gap it must close reflects not only i's investment but also the data-driven quality advantage that the same investment unlocks. On the secondary market, by contrast, it retreats: it is less willing to subsidize adoption as it is less likely to compensate for losses —it raises its secondary price. The ecosystem structure thus amplifies the strategic importance of quality, linking competition for digital users to the balance of profits across the two markets.

Lemma 1 (Data-induced distortion in price sensitivity)**.** *The presence of the secondary data-rich market makes prices respond to quality differences in opposite ways across the two markets. In the primary market, each firm's price rises in its own quality and falls in its rival's, more strongly than in the benchmark:*

$$\frac{\partial p_{i_1}}{\partial q_i} = -\frac{\partial p_{i_1}}{\partial q_j} = \tfrac{3}{7} > \tfrac{1}{3}.$$

In the secondary market the pattern reverses—each firm's price falls in its own quality and rises in its rival's:

$$\frac{\partial p_{i_2}}{\partial q_i} = -\frac{\partial p_{i_2}}{\partial q_j} = -\tfrac{1}{7}.$$

Proof. Immediate from $p_{i_1} = 1 + \frac{3}{7}(q_i - q_j)$ against the benchmark $p_i^b = 1 + \frac{1}{3}(q_i - q_j)$, and $p_{i_2} = \rho - 1 - \frac{1}{7}(q_i - q_j)$. □

Substituting the equilibrium prices into the profit function of firm A yields:

$$\begin{aligned}\pi_A(q_A, q_B) = {} & p_{A_1}(q_A, q_B) \cdot \tilde{x}\big(p_{A_1}(q_A, q_B),\, p_{B_1}(q_A, q_B),\, p_{A_2}(q_A, q_B),\, p_{B_2}(q_A, q_B),\, q_A, q_B\big) \\ & + p_{A_2}(q_A, q_B) \cdot \tilde{y}\big(p_{A_2}(q_A, q_B),\, p_{B_2}(q_A, q_B)\big) \\ & - \frac{q_A^2}{2\eta}.\end{aligned}$$

Firm A's investment choice affects profits through several channels. Using the envelope property from the pricing stage, the derivative of profit with respect to q_A can be written as

$$\frac{\partial \pi_A}{\partial q_A} = p_{A_1}\left(\frac{\partial \tilde{x}}{\partial p_{B_1}}\frac{\partial p_{B_1}}{\partial q_A} + \frac{\partial \tilde{x}}{\partial q_A}\right) + \underbrace{(p_{A_2} + p_{A_1})\left(\frac{\partial \tilde{y}}{\partial p_{B_2}}\frac{\partial p_{B_2}}{\partial q_A}\right)}_{\text{ecosystem marginal return}} - q_A/\eta. \tag{8}$$

Beyond the marginal cost, this expression highlights two distinct marginal returns to quality investment, which pull investment in opposite directions.

The first term is the primary–market marginal return. Improving quality directly raises primary-market demand, but it also induces a strategic response from the rival, who lowers its price and thereby recovers part of firm A's demand—the standard strategic effect in Hotelling models with endogenous quality. Here, however, this effect is stronger than in the benchmark. By Lemma 1, primary-market prices respond more sharply to quality differences, because a quality advantage is now reinforced by the data it lets firm A accumulate. The rival therefore cuts its price more aggressively in response to a marginal quality increase, which lowers the marginal profitability of investment relative to the benchmark.

The second term is the ecosystem marginal return, and it works in the opposite direction. Each additional consumer in the secondary market is worth not only its direct revenue p_{A_2} but also the value its data generate in the primary market, equal to p_{A_1}; this is why the channel is multiplied by $(p_{A_2} + p_{A_1})$. The return is positive because the rival softens its secondary-market pricing in response to higher quality (Lemma 1 stressed that $\partial p_{B_2}/\partial q_A \geq 0$): a quality lead depresses the rival's primary price, so the primary sales are less profitable, and it subsidizes less for digital users. Firm A thus captures them, and the data they supply, at a higher margin. At the symmetric equilibrium the term simplifies to $p_{A_2} + p_{A_1} = \rho$, so this ecosystem return is positive and grows with the size of the secondary market.

Which of these two effects dominates determines whether equilibrium quality is higher or lower than in the benchmark. In the symmetric equilibrium, the quality levels under competing ecosystems are

$$q_A^* = q_B^* = \frac{(4+\rho)\eta}{14},$$

provided the cost-efficiency is not too large $\eta \leq 7$. I retain this new parameter restriction on cost-efficiency for the remainder of the paper.

Proposition 1 (Quality comparison with benchmark). *In the symmetric equilibrium, the ecosystems invest less than in the benchmark if and only if $\rho < \frac{2}{3}$, and more otherwise.*

Proof. Immediate from $q_i^b = \eta/3$ and $q_i^* = \frac{(4+\rho)\eta}{14}$. □

The corresponding symmetric equilibrium outcomes are

$$p_{A_1}^* = p_{B_1}^* = 1, \qquad p_{A_2}^* = p_{B_2}^* = \rho - 1,$$

$$x^* = \frac{1}{2}, \qquad y^* = \frac{\rho}{2},$$

$$\pi_A^* = \pi_B^* = \frac{1}{2} - (1-\rho)\frac{\rho}{2} - \frac{\eta[(4+\rho)]^2}{2 \times 14^2}, \quad CS^* = v_1 + \frac{1}{14}\eta(\rho+4) + \frac{1}{4}(\rho(-5\rho+4v_2+6)-5).$$

Comparison of quality with the benchmark highlights the central role of the secondary–market size ρ. When ρ is small, firms must heavily subsidize digital adoption, so that secondary market prices are negative and the total value $(p_{A_2}+p_{A_1})$ of a digital user is limited. The ecosystem channel then provides weak incentives to invest in quality. At the same time, the mere presence of the secondary market makes primary–market prices more sensitive to quality differences, which intensifies price competition. Firms therefore reduce investment with respect to the benchmark case.

As ρ increases, competition in the secondary market relaxes and secondary market prices rise. The term $(p_{A_2}+p_{A_1})$ becomes larger, which increases the ecosystem marginal return. This second effect eventually dominates and overturns the result leading to higher equilibrium quality than the benchmark case.

The next proposition highlights how the rest of the outcomes differ from the benchmark case.

Proposition 2 (Outcomes comparison with Benchmark). *In the symmetric equilibrium:*

1. *Secondary-market prices are strictly below primary-market prices and negative, while primary-market prices coincide with the benchmark.*

2. *Profits exceed the benchmark if and only if*

$$\rho < \frac{2\left(147 + 6\eta - 7\sqrt{\eta^2 - 16\eta + 441}\right)}{3\,(196 - \eta)},$$

and fall below it otherwise.

3. *Consumer surplus is lower than the benchmark if and only if*

$$\rho < \frac{1}{105}\left[3\eta + 42v_2 + 63 \;-\; \sqrt{(3\eta + 42v_2 + 63)^2 - 420\,\eta}\right],$$

and higher otherwise.

Proof. See appendix. □

The intuition behind these results are is follows.

Prices reflect the fact that firms are willing to subsidize adoption in the secondary market to accumulate data that enhances primary-market demand. When the secondary market size is small, the subsidy is larger as competition is fierce for the few consumers there and secondary prices can become negative, highlighting the strategic nature of the digital secondary market as a loss leader.

Consider now profits. The effect of the secondary market on profits is driven by two opposing forces: digital losses and higher investment.

The first term is the benchmark Hotelling profits from the primary–market profit. The second term captures losses from the secondary market, since $p^*_{A_2} = \rho - 1 < 0$. This term is U-shaped in ρ: enlarging the secondary market initially increases these losses, but for larger ρ the losses shrink as digital prices rise. The last term is the cost of quality investment, which increases monotonically in ρ because equilibrium quality rises.

When $\rho \approx 0$, profits are close to the benchmark Hotelling setting and are reduced by smaller quality investment than the benchmark case. Increasing the size of the secondary market mainly increases secondary-market losses, while raising quality investment costs. Profits therefore fall and can drop below the benchmark case. For larger ρ, secondary-market losses begin to shrink. However, investment costs keep rising since quality continues to increase. But the first effect dominates and profits increase. They still remain below the benchmark because the higher investment cost more than offsets the reduction in digital losses compared to the benchmark.

This explanation suggests that the competitive degree on the secondary market may affect this result. Section 5 indeed shows that a lower degree of competition in the secondary market can make profits become higher than the benchmark for sufficiently high market size.

Turning to consumer surplus. When the secondary market size is small, adoption of digital products is heavily subsidized, but this market is small while primary-market competition remains intense and investment has decreased, leading to lower consumer surplus relative to the benchmark. As the market size of the secondary market increases, negative secondary-market prices for a larger number of consumers and enhanced quality investment improve total value for consumers. Once the size is sufficiently great, consumer surplus surpasses the benchmark.

5 The degree of competition

In this section, I study how the intensity of competition in the secondary market affects our main result regarding the opposing forces between secondary-market losses and quality investment, which jointly generate the U-shaped relationship between profits and market size.

To this end, I introduce horizontal differentiation in the secondary market. Consumers located on the secondary market now incur a transportation cost $t \geq 1$ when purchasing from a firm, which includes our baseline where $t = 1$. This parameter captures the degree of product differentiation and therefore the degree of competition: a higher t corresponds to weaker competition.

The introduction of differentiation modifies both the equilibrium secondary-market prices and the equilibrium quality choices, which now depend explicitly on the competitive parameter t. However, they do not modify the primary-market prices. In the symmetric equilibrium, I obtain

$$p^t_{A_2} = p^t_{B_2} = t\rho - 1, \qquad q^t_A = q^t_B = \frac{\eta[(6+\rho)t - 2]}{18t - 4}.$$

The qualitative mechanisms identified in the baseline model remain at work. But I can now show more formally that a reduction in competition (an increase in t) mitigates the firms' losses on the secondary market. At the same time, weaker competition reduces incentives to invest in quality. Taken together, these two effects imply that profits increase more strongly with market size than in the baseline model without differentiation. In particular, profits may now exceed the benchmark profit level.

This result is summarized in the following proposition.

Proposition 3. *Suppose that consumers on the secondary market incur a transportation cost $t \geq 1$. In the symmetric equilibrium, firms' profits exceed the benchmark level whenever $\rho \in (\underline{\rho}(t, \eta), 1]$, where*

$$\underline{\rho}(t,\eta) = \frac{2\left(3\left[(9t-2)^2 + \eta t(3t-1)\right] + (9t-2)\sqrt{9(9t-2)^2 - 2\eta t(9t-1) + \eta^2 t^2}\right)}{3t\left[\,4(9t-2)^2 - \eta t\,\right]}.$$

Another noteworthy implication of the effect of reduced competition in the secondary market concerns consumer surplus. Recall that, in the baseline model, an increase in market size eventually increases consumer surplus above its benchmark value essentially due to the higher quality provision. With horizontal differentiation in the secondary market, this mechanism is reinforced by reduced subsidies on the secondary-market size. As shown in Proposition 3, weaker competition reduces secondary-market losses and dampens quality investment, which allows firms' profits to rise above the benchmark level for a range of parameters. Importantly, I find that over this same range, consumer surplus actually continues to exceed its benchmark value.

Hence, there exists a non-empty set of parameters for which both firms and consumers are strictly better off relative to the benchmark case. Figure 1 provides an illustration. Reduced competition in the secondary market therefore generates a Pareto improvement compared to the benchmark environment. This observation is formalized in the following proposition.

Proposition 4. *There exists a non-empty set of parameters* (ρ, t) *for all* $1 \leq \eta \leq 7$ *such that, in the symmetric equilibrium with transportation cost on the secondary market,*

1. *firms' profits strictly exceed the benchmark level, and*
2. *consumer surplus strictly exceeds its benchmark level.*

In this region, both firms and consumers are strictly better off relative to the benchmark case without secondary market.

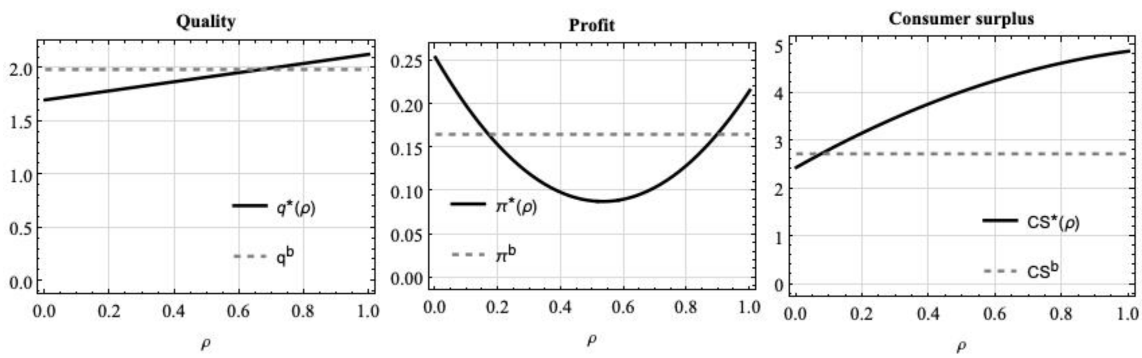

Figure 1: *Comparative statics of equilibrium outcomes under competing ecosystems.* Equilibrium outcomes are plotted as functions of the size of the digital market $\rho \in (0, 1]$ for $v_1 = v_2 = 2$, $\eta = 6$ and $t = 1.2$. Benchmark levels are shown by horizontal dashed lines.

The main result of this section runs counter to the conventional view that stronger competition between digital ecosystems necessarily benefits consumers. I show that this need not be the case. In the baseline model, profits remain below the benchmark even as the digital market matures because intense competition for data leads firms to offer substantial subsidies and engage in excessive quality investment. Introducing higher horizontal differentiation softens competition in the secondary market. This reduces losses incurred in that market and weakens firms' incentives to overinvest in quality. Both effects increase equilibrium profits and generate a parameter region in which profits exceed the benchmark level for great secondary market size.

6 Endogenous expansion choice

The baseline section shows that if firms both expand in the secondary market, they may both lose profits relative to a benchmark in which they do not expand. One could argue that this situation arises as a consequence of both firms bilaterally expanding when the market size is small, and that once the digital market reaches a sufficient size, both firms start losing the benefits of such expansion.

I now ask whether the competitive configuration arises endogenously, and show that each firm has a unilateral incentive to encroach on the secondary market irrespective of its size, so that the symmetric ecosystem structure can emerge even when it leaves both firms worse off.

To do that, I only append one stage to the baseline model: before investing in quality, each firm decides whether to operate a digital arm, incurring a fixed cost $f \geq 0$ that reflects the organizational and operational investment required to develop and maintain a dedicated digital unit. A firm that does not expand is active only in the primary market.

Suppose firm A expands and firm B does not. Because B does not operate in the secondary market, firm A serves all of it: it sets the highest price compatible with full coverage, $p_{A_2}^{E_A} = v_2 - \rho$, obtaining $d_A = \rho$ and $d_B = 0$. With similar computations as in the baseline model (detailed in the Appendix), the equilibrium outcomes are

$$q_A^{E_A} = \frac{\eta\big(9 - 2\eta + 3\rho\big)}{3(9 - 2\eta)} \quad \& \quad q_B^{E_A} = \frac{\eta\big(9 - 2\eta - 3\rho\big)}{3(9 - 2\eta)}, \tag{9}$$

$$p_{A_1}^{E_A} = 1 + \frac{3\rho}{9 - 2\eta} \quad \& \quad p_{B_1}^{E_A} = 1 - \frac{3\rho}{9 - 2\eta} \quad \& \quad p_{A_2}^{E_A} = v_2 - \rho, \tag{10}$$

$$\pi_A^{E_A} = \frac{(9 - \eta)\big(9 - 2\eta + 3\rho\big)^2}{18(9 - 2\eta)^2} + v_2\rho - \rho^2 \quad \& \quad \pi_B^{E_A} = \frac{(9 - \eta)\big(9 - 2\eta - 3\rho\big)^2}{18(9 - 2\eta)^2}. \tag{11}$$

The entrant secures a data advantage that boosts its quality investment and depresses the rival's, so that A captures more than half of the primary market. These expressions describe an interior configuration in which the rival is not driven out of the primary market whenever $x_B^{E_A} = \frac{1}{2} - \frac{3\rho}{2(9-2\eta)} \geq 0$. Since it must hold for every $\rho \in (0, 1]$, it implies $\eta \leq 3$; it also guarantees $9 - 2\eta > 0$ and so positive quality choices.

Computing the symmetric outcome when B expands and A does not yields the 2×2

expansion game in Table 1, where $\pi_i^{E_j}$ denotes the profit of firm i that abstains while its rival j expands.

		Firm B	
		Expand	**Not Expand**
Firm A	**Expand**	$\pi_A^* - f\ ,\ \pi_B^* - f$	$\pi_A^{E_A} - f\ ,\ \pi_B^{E_A}$
	Not Expand	$\pi_A^{E_B}\ ,\ \pi_B^{E_B} - f$	$\pi_A^b\ ,\ \pi_B^b$

Table 1: Expansion game profits

A firm prefers to expand when its rival does not if and only if the fixed cost is below

$$f^{EN}(\rho) := \pi_A^{E_A}(\rho) - \pi_A^b,$$

and prefers to expand when its rival also expands if and only if it is below

$$f^{EE}(\rho) := \pi_A^*(\rho) - \pi_A^{E_B}(\rho).$$

These two thresholds, compared with f, fully determine the equilibrium. The key property is that they are ordered: unilateral expansion is strictly more attractive than joint expansion,

$$0 < f^{EE}(\rho) < f^{EN}(\rho). \tag{12}$$

The reason is that a sole expansionist monopolizes the secondary segment and collects the rent, together with a primary-market data advantage, whereas two expansionists compete that rent away to a loss. A firm gains more from expanding when its rival does not. [7]

Computing best responses with the ordering of Equation (12) gives the equilibrium structure of the expansion game.

Proposition 5 (Equilibria of the expansion game)**.** *Under previous conditions and the new more stringent condition that $\eta \leq 3$, the entry game has:*

1. *if $f < f^{EE}(\rho)$: a unique equilibrium (E, E) — both firms enter;*
2. *if $f^{EE}(\rho) < f < f^{EN}(\rho)$: two equilibria (E, N) and (N, E) — exactly one firm enters;*

[7] The ordering (12) holds for ρ bounded away from zero. As $\rho \to 0$ the secondary market vanishes, so $f^{EN} \to 0$ while $f^{EE} \to \frac{13\eta}{882} > 0$ because π_A^* does not converge to π_A^b, the sharper price-sensitivity persists, and the inequality reverses on a negligible interval $(0, \rho^*(\eta))$ with $\rho^*(\eta) \leq 0.018$ for $\eta \leq 3$, $v_2 = 2$. I therefore restrict attention to $\rho \in [\underline{\rho}_0, 1]$ with $\underline{\rho}_0 = 0.02$, on which $0 < f^{EE} < f^{EN}$.

3. *if $f > f^{EN}(\rho)$: a unique equilibrium (N, N) — neither firm enters.*

Moreover, the mutual-entry equilibrium of case 1 is Pareto-dominated by mutual non-entry if and only if $\pi_A^*(\rho) < \pi_A^b$, equivalently $\rho > \rho_\pi$, where ρ_π is the profit threshold of Proposition 2.

Proof. See Appendix. □

Three implications are worth emphasizing.

First, in the absence of a fixed cost ($f = 0$, hence $f < f^{EE}$ by (12)), both firms expand: encroaching on the secondary market is a dominant strategy. This formalizes the unilateral incentive to build a digital arm in order to secure the data advantage.

Second, this mutual entry is a *prisoner's dilemma* precisely when the digital market is large enough that data competition depresses profits below the benchmark, $\rho > \rho_\pi$: each firm expands, yet both would be strictly better off abstaining. Notably, the dilemma sets in at exactly the threshold ρ_π identified in Section 4. For $\rho < \rho_\pi$ both firms still expand, but mutual entry is jointly profitable, so there is no dilemma—merely the efficient emergence of competing ecosystems.

Third, when the fixed cost is intermediate, $f \in (f^{EE}, f^{EN})$, the symmetric firms cannot both profitably operate a digital arm, and the only equilibria are asymmetric: exactly one firm becomes the digital incumbent. The multiplicity is over which firm expands, since the two outcomes are symmetric. This somehow reconnects to the literature on ecosystems, which studies dominant firms expanding into secondary markets (Condorelli and Padilla, 2024). In our setting, however, a firm becomes dominant on the secondary market only because it has expanded while its rival has not, and it is too costly for the latter to follow.

Finally, both entry thresholds respond to the size of the secondary market, but in different ways.

Proposition 6 (Entry incentives and market size)**.** *Under the maintained conditions, with $v_2 = 2$:*

1. *$f^{EN}(\rho)$ is strictly increasing in ρ:*

$$\frac{\partial f^{EN}}{\partial \rho} = \frac{(9-\eta)\,(9-2\eta+3\rho)}{3(9-2\eta)^2} + \big(v_2 - 2\rho\big) > 0.$$

2. *$f^{EE}(\rho)$ is strictly convex (U-shaped) in ρ, with*

$$\frac{\partial f^{EE}}{\partial \rho} = \rho - \tfrac{1}{2} - \tfrac{\eta(\rho+4)}{196} + \frac{(9-\eta)\,(9-2\eta-3\rho)}{3(9-2\eta)^2},$$

so that f^{EE} decreases for $\rho < \rho^{EE}(\eta)$ and increases for $\rho > \rho^{EE}(\eta)$; the turning point $\rho^{EE}(\eta)$ is small ($\rho^{EE}(1) \approx 0.17$) and is non-positive once $\eta \gtrsim 2.5$, in which case f^{EE} increases throughout.

Proof. See Appendix. □

The incentive to be the unique firm on the digital market rises monotonically with its size: a larger secondary segment enlarges the monopoly rent the expansionist can appropriate, and although unilateral entry distorts primary-market quality choices (the expansionist over-invests, the rival under-invests, intensifying price competition), this effect never offsets the rent. The incentive to 'co-expand', by contrast, is U-shaped in digital market size: it inherits the U-shape of competing-ecosystem profits π_A^* from Section 4—falling while data competition intensifies, then rising once the segment is large enough to be worth serving— moderated by the fact that abstention becomes increasingly costly as the rival's data advantage grows. Consequently the range of fixed costs for which only one firm expands, (f^{EE}, f^{EN}), generally widens with the size of the digital market.

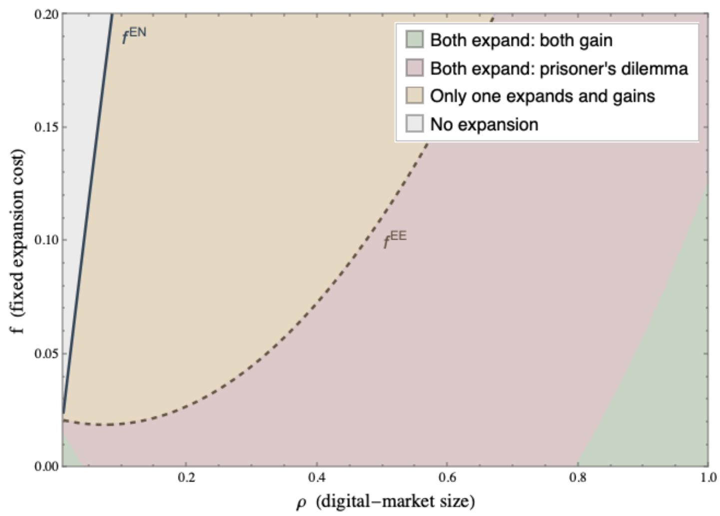

Figure 2: Firms gain and losses regions with respect to the fixed cost and digital market size (ρ, f) with the thresholds f^{EE} (dashed) and f^{EN} (solid): both firms expand (below f^{EE}), one expands (between), or neither expands (above f^{EN}). In the mutual-entry region firms beat the benchmark only below $f = \pi_A^* - \pi_A^b$ (green); elsewhere mutual expansion is a prisoner's dilemma (red). Parameters: $\eta = 2$, $v_2 = 2$, $t = 1.3$.

Degree of competition. Reintroducing the degree of competition of Section 5 sharpens this picture. With sufficient product differentiation in the secondary market, the mutual-expansion profit exceeds the benchmark even for a large digital market (Proposition 3), while expansion remains a dominant strategy at low fixed costs. There then exists a non-empty set of parameters—a sufficiently large and sufficiently differentiated digital market—in which both firms expand in equilibrium and each earns strictly more than under no expansion. Differentiation thus converts the prisoner's dilemma of the baseline, where endogenous expansion leaves both firms worse off, into a mutually beneficial outcome: precisely the large-ρ region that is a dilemma at $t = 1$ becomes one in which the endogenous emergence of competing ecosystems makes both firms better off. Figure 2 illustrates our findings.

7 Discussions of policy implications

The analysis highlights two main channels through which policy can affect welfare in the presence of competing digital ecosystems.

7.1 Expanding the digital market: adoption, training, and infrastructure

A point of attention is that when the digital market is small, consumers are often worse off due to lower quality investment efforts by firms. A first policy lever therefore consists in increasing the size of the digital market, for instance by promoting the adoption of digital tools among farmers through training programs, information campaigns, or adoption subsidies. When the digital market is sufficiently large, the ecosystem marginal return dominates and both consumer surplus and product quality increase above the benchmark without a digital market. Hence, policies that facilitate digital adoption can mitigate the adverse effects associated with low levels of digital market size and benefit consumers.

Concrete examples of such policies in the agricultural sector include the European Commission's integration of digitalisation strategies into the Common Agricultural Policy (CAP) 2023–2027 Strategic Plans, which explicitly support investments in digital infrastructure such as broadband, precision farming technologies, and advisory services for digitalisation (European Commission, 2025). These measures are designed to improve connectivity, digital skills, and the uptake of advanced technologies among farmers, thus expanding the digital segment in agriculture. Evidence from the Joint Research Centre shows that adoption rates of digital tools are higher where connectivity and training are available, while high costs and limited digital skills remain key barriers to broader uptake (European Commission Joint Research Centre, 2025). The analysis is based on farm survey data from 1 444 respondents in nine EU Member States.

7.2 Regulating entry costs to operate in digital sector

A second policy lever operates on firms' incentives to expand and operate in the digital market. This can be achieved by altering the fixed cost of operating digital platforms depending on the expected size of the secondary market, for example through regulation, compliance requirements, or targeted taxation. In the model, this corresponds to a change in fixed cost, which can delay or limit the transition toward competing ecosystems. Because consumer surplus is higher than benchmark only for intermediate or high levels of digital market size, moderating entry at early stages of market development can prevent consumer surplus reduction.

A relevant policy example is the EU Data Act (Regulation (EU) 2023/2854), which introduces cross-sectoral legal obligations for data holders and providers of data processing services to grant users access to the data they generate and, in some cases, to share data with third parties on fair and non-discriminatory terms (European Union, 2023). It also requires providers of cloud and related services to ensure portability and interoperability and to remove barriers to switching providers. These obligations impose significant compliance and technical costs, from designing APIs and data export interfaces to adapting contractual and pricing frameworks, which effectively increase the fixed cost of operating digital platforms. In our model, such regulatory interventions influence firms' entry decisions by altering the cost structure associated with competing digital ecosystems.

7.3 Regulating ecosystem-building acquisitions and product differentiation

These insights also resonate with recent policy discussions on digital mergers and acquisitions, notably in *The Future of European Competitiveness* by Mario Draghi, which argues that Europe's competitiveness gap stems in part from underinvestment in digital technologies, data infrastructure, and AI. The report calls for policies that strengthen Europe's digital capacity and suggests that merging firms should be allowed to justify a merger by demonstrating that it will increase innovation, provided that such a defence is supported by sufficiently concrete and verifiable evidence (Draghi, 2024).

In practice, mergers and partnerships in digital agriculture can be interpreted as substantial ecosystem-building investments that effectively act as fixed costs in our model. For example, Bayer's acquisition of The Climate Corporation in 2013 provided the technological foundation for FieldView[8]; Syngenta's acquisition of FarmShots enhanced its remote sensing and data analytics capabilities[9]; and John Deere's purchase of Blue River Technology integrated machine vision and AI into its digital equipment ecosystem[10]. These transactions involve large upfront investments in data infrastructure, technological integration, and organizational capabilities that shape the structure and competitiveness

[8] "Monsanto buys Climate Corporation for about $1.1 billion." Available at: https://www.farm-equipment.com/articles/9462-monsanto-to-acquire-the-climate-corporation.

[9] 'Syngenta acquires FarmShots to boost remote sensing capabilities." Available at: https://globalaginvesting.com/syngenta-acquires-farmshots/.

[10] TechCrunch (2017), "John Deere buys Blue River Technology for $305M." Available at: https://www.cnbc.com/2017/09/06/deere-is-acquiring-blue-river-technology-for-305-million.html.

of digital ecosystems in agriculture. In the context of our model, the welfare impact of these investments depends critically on the stage of digital market development: in early stages, they may harm consumers, while in mature markets, they may benefit them.

Finally, our analysis highlights that authorities should pay close attention to the degree of competition and product differentiation in digital markets. Somewhat counterintuitively, a greater degree of differentiation between digital ecosystems can improve welfare. By softening direct price competition for data while preserving incentives to invest in quality, differentiation mitigates the distortions associated with intense rivalry for data and can lead to higher consumer surplus and profits in equilibrium.

This insight is particularly relevant in the agricultural sector, where digital technologies are inherently heterogeneous and often embedded in highly specific agronomic environments. In particular, Decision Support Systems (DSS) are frequently specialised by crop, variety, or production context, reflecting the underlying heterogeneity of biological risks and farming practices. The digital agronomy catalogue published by Phyteis in its 2025 Digital Agronomy Directory illustrates this pattern, showing that most agricultural digital tools are not generic platforms but targeted solutions tailored to specific crops and problems, including cereal disease monitoring, vineyard pest management, maize wireworm risk, and canola protection[11].

7.4 Pricing regulation: effects of a no below-cost constraint

In our equilibrium outcomes, firms may optimally set negative prices on the secondary market to attract consumers. Suppose instead that regulation imposes a no-below-cost pricing constraint, which formally requires $p_{i2} \geq 0 \quad \text{for all } i$.

The constraint applies at the pricing stage. From the price expressions in Equation (7), the price of firm i is (strictly) positive only if the quality advantage of rival j is sufficiently large $q_j > 7(1-\rho) + q_i$. In that case, the rival's price must be zero because constrained by the no-below cost rule. This configuration generates asymmetric outcomes. I do not study it further because our focus is on finding a symmetric equilibrium to have neat comparison with our two previous symmetric equilibria.

By contrast, when quality differences are sufficiently small,

$$|q_i - q_j| \leq 7(1-\rho),$$

both unconstrained prices would be negative, and the constraint implies $p_{i2} = 0$ for both firms. This region includes the symmetric configuration and is therefore our focus.

In this case, firms anticipate that secondary-market prices will be constrained at zero. As a result, marginal changes in quality no longer affect secondary-market prices or

[11]"Phyteis dévoile son répertoire Agronomie digitale 2025." Available at: https://phyteis.fr/actualites/phyteis-devoile-son-repertoire-agronomie-digitale-2025/.

market shares. Since neither investment nor pricing decisions influence outcomes in the secondary market, each firm expects to serve half of that market. Consequently, firms choose the same quality level as in the benchmark case and earn identical profits, as the secondary market generates neither revenues nor losses.

Consumers, however, benefit from the data-driven quality improvements provided by firms at no cost. Consumer surplus therefore increases with the size of the secondary market. Overall, a ban on below-cost pricing benefits consumers while leaving firms no better off and providing no additional incentives to invest in quality. However, and importantly, it also eliminates the possibility of achieving an outcome in which both firms and consumers benefit from the expansion of the secondary market.

8 Robustness to alternative specifications

Spillover from primary R&D to secondary-product quality. Suppose that R&D investment in the primary product spills over only partially to the secondary-product quality. Formally, replace the secondary-product value v_2 by $v_2 + s\,q_i$ for $i \in \{A, B\}$, with $s \in [0, 1]$ parametrising the strength of the spillover (the baseline corresponds to $s = 0$). Because quality now raises demand in both markets, the quality-stage second-order condition tightens to $\eta < \bar{\eta}(s) := 7/(1 + s + s^2)$, which decreases from $\bar{\eta}(0) = 7$ to $\bar{\eta}(1) = 7/3$; we therefore evaluate this extension on the s-dependent admissible set $\eta \in [1, \bar{\eta}(s))$, which contains the baseline restriction at $s = 0$ and narrows as the spillover strengthens. On this set, all our qualitative results survive and are actually enhanced: the conclusions of Proposition 2 hold for the baseline model and those of Proposition 4 hold with a secondary-market transport cost, albeit at different numerical thresholds. The only systematic change concerns the response of quality investment to the size of the secondary market: with spillover, firms invest less when the secondary market is small and more when it is large, so that equilibrium quality becomes steeper in ρ. This is intuitive: quality now affects both market demands, so the high-ρ incentive to overinvest in quality in order to attract more secondary buyers is amplified.

Reduced data-driven quality effect δ. Suppose the data-driven quality channel is scaled by $\delta \in [0, 1]$, so the primary-market utility becomes $v_1 + q_i + \delta d_i - p_{i_1} - x$; the baseline corresponds to $\delta = 1$. This parameter can represent a discount factor for the delayed arrival of the data-driven quality, or frictions in the data accumulation process. The equilibrium exists, is symmetric, and varies continuously in δ, with $p_1^* = 1$ and $p_2^* = \rho t - \delta$. The second-order condition here is $\eta \leq 9 - 2\delta^2/t$, which is at least 7 for every $\delta \in [0, 1]$ and $t \geq 1$; the maintained restriction $\eta \leq 7$ therefore continues to guarantee a concave objective. The qualitative results are robust. Proposition 2 holds unchanged when the data channel is strong enough ($\delta > \delta^*(\eta)$), and weakens only mildly otherwise:

as δ falls past $\delta^*(\eta)$, the small-market region in which profits exceed the benchmark is joined by an additional large-market region, so the qualitative message that small secondary markets can sustain above-benchmark profits continues to hold even though the exact "if and only if" no longer does. In the limit $\delta = 0$ the data channel vanishes: quality reverts to the benchmark, the secondary market becomes simply an additional profitable segment, and profits exceed the benchmark for all market sizes.

9 Conclusion

I study competition between symmetric firms that operate as ecosystems across a primary market and a secondary data-rich digital market. Because digital adoption feeds back into product quality in the primary market, the size of the digital segment reshapes firms' incentives to invest ex ante. When adoption of the digital product is limited, firms invest less than in a benchmark without digital services; when adoption is sufficiently large, data complementarities overturn this effect and investment rises above the benchmark. Profits and consumer surplus move in opposite directions as the digital market expands. However, when digital products are sufficiently differentiated, profits and consumer surplus can both rise above benchmark levels, so that the emergence of competing ecosystems benefits both firms and consumers.

Allowing for endogenous entry reveals a prisoner's dilemma: even when bilateral digital encroachment reduces profits, each firm has a unilateral incentive to expand the digital market to secure the data advantage. The fixed cost of operating digital services is therefore central for equilibrium outcomes. Low entry costs can induce firms to expand "too early," when digital adoption is still limited, thereby reducing consumer surplus.

These conclusions survive partial R&D spillovers to digital quality and a weaker data-driven quality channel.

The present paper therefore provides a useful framework for evaluating policies that affect digital adoption, the technical requirements for operating digital services, pricing constraints, and ecosystem formation in traditional industries undergoing digital transition.

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APPENDIX

Computations for benchmark equilibrium outcomes. I first derive a benchmark in which firms operate exclusively in the primary market, so that product quality is determined solely by ex ante investment. I use backward induction to solve for the subgame perfect Nash equilibrium. Consumer demand follows the standard Hotelling computations whereby the consumer indifferent between buying from firm A and buying from firm B, denoted $\tilde{x}^b$, is such that:

$$v_1 + q_A - p_{A_1} - \tilde{x}^b = v_1 + q_B - p_{B_1} - (1 - \tilde{x}^b).$$

This yields a marginal consumer location

$$\tilde{x}^b(p_{A_1}, p_{B_1}) = \frac{1}{2} + \frac{1}{2}(q_A - q_B + p_{B_1} - p_{A_1}).$$

Firm profits, omitting sunk investment costs, are $\Pi_A(p_{A_1}, p_{B_1}) = p_{A_1}\tilde{x}^b(p_{A_1}, p_{B_1})$ and $\Pi_B(p_{A_1}, p_{B_1}) = p_{B_1}(1 - \tilde{x}^b(p_{A_1}, p_{B_1}))$, from which I get the standard first-order conditions:

$$\frac{\partial \Pi_A}{\partial p_{A_1}} = 0 \Leftrightarrow \frac{1}{2} + \frac{1}{2}(q_A - q_B + p_{B_1} - p_{A_1}) - \frac{1}{2}p_{A_1} = 0, \tag{13}$$

$$\frac{\partial \Pi_B}{\partial p_{B_1}} = 0 \Leftrightarrow \frac{1}{2} - \frac{1}{2}(q_A - q_B + p_{B_1} - p_{A_1}) - \frac{1}{2}p_{B_1} = 0. \tag{14}$$

And the second-order derivatives give

$$\frac{\partial^2 \Pi_A}{\partial p_{A_1}^2} = -1, \tag{15}$$

$$\frac{\partial^2 \Pi_B}{\partial p_{B_1}^2} = -1. \tag{16}$$

The second-order condition is satisfied as second-order derivatives are negative and solving the first-order conditions for equilibrium prices (using symmetry) gives

$$p_A^b(q_A, q_B) = 1 + \frac{1}{3}(q_A - q_B) \quad \text{and} \quad p_B^b(q_A, q_B) = 1 - \frac{1}{3}(q_A - q_B).$$

Substituting these equilibrium prices into profit functions, and now incorporating the sunk investment cost $C(q_i) = q_i^2/(2\eta)$, yields

$$\pi_A(q_A, q_B) = \frac{(q_A - q_B + 3)^2}{18} - \frac{q_A^2}{2\eta} \quad \text{and} \quad \pi_B(q_A, q_B) = \frac{(q_B - q_A + 3)^2}{18} - \frac{q_B^2}{2\eta}.$$

The first-order conditions yield

$$\frac{\partial \pi_A}{\partial q_A} = 0 \Leftrightarrow \frac{2(q_A - q_B + 3)}{18} - \frac{q_A}{\eta} = 0 \tag{17}$$

$$\frac{\partial \pi_B}{\partial q_B} = 0 \Leftrightarrow \frac{2(q_B - q_A + 3)}{18} - \frac{q_B}{\eta} = 0. \tag{18}$$

The second-order derivatives yield

$$\frac{\partial^2 \pi_A}{\partial q_A^2} = \frac{1}{9} - \frac{1}{\eta} \tag{19}$$

$$\frac{\partial^2 \pi_B}{\partial q_B^2} = \frac{1}{9} - \frac{1}{\eta}. \tag{20}$$

The second-order condition is satisfied as second-order derivatives are negative whenever $\eta \leq 9$, which I assume holds throughout. Solving the first-order conditions for quality investments gives a symmetric outcome:

$$q_A^b = q_B^b = \frac{\eta}{3}.$$

At this benchmark, since quality investments are symmetric, equilibrium prices and quantities are symmetric,

$$p_A^b = p_B^b = 1,$$

and demand is equally split between the firms

$$\tilde{x}^b = 1/2.$$

The corresponding firm profits are

$$\pi_A^b = \pi_B^b = \frac{(0+3)^2}{18} - \frac{(\eta/3)^2}{2\eta} = \frac{9}{18} - \frac{\eta}{18} = \frac{9-\eta}{18}.$$

Consumer surplus is obtained by integrating the net utility of each consumer over the unit line. Firm A serves consumers on $[0, \tilde{x}^b]$ and firm B serves consumers on $[\tilde{x}^b, 1]$, so

$$CS^b = \int_0^{\tilde{x}^b} \left(v_1 + q_A^b - p_A^b - x\right) dx + \int_{\tilde{x}^b}^1 \left(v_1 + q_B^b - p_B^b - (1-x)\right) dx.$$

By substitution ($\tilde{x}^b = \frac{1}{2}$, $q_A^b = q_B^b = \frac{\eta}{3}$, $p_A^b = p_B^b = 1$), both integrals coincide and

$$CS^b = 2 \int_0^{1/2} \left(v_1 + \tfrac{\eta}{3} - 1 - x\right) dx.$$

Evaluating the integral,

$$CS^b = 2\left[\left(v_1 + \tfrac{\eta}{3} - 1\right)x - \frac{x^2}{2}\right]_0^{1/2} = 2\left[\frac{1}{2}\left(v_1 + \tfrac{\eta}{3} - 1\right) - \frac{1}{8}\right] = v_1 + \frac{\eta}{3} - 1 - \frac{1}{4},$$

so that

$$CS^b = v_1 + \frac{\eta}{3} - \frac{5}{4}.$$

□

Computations for baseline model equilibrium outcomes. I use backward induction to solve for the subgame perfect Nash equilibrium. In the secondary market, the indifferent consumer satisfies

$$v_2 - p_{A_2} - y = v_2 - p_{B_2} - (\rho - y),$$

which implies

$$\tilde{y}(p_{A_2}, p_{B_2}) = \frac{\rho}{2} + \frac{1}{2}(p_{B_2} - p_{A_2}).$$

In the primary market, the indifferent consumer now satisfies

$$v_1 + q_A + d_A - p_{A_1} - x = v_1 + q_B + d_B - p_{B_1} - (1 - x),$$

I suppose consumers have rational expectations and anticipate that data generated in the secondary market equals realized demand:

$$d_A = \tilde{y}, \qquad d_B = \rho - \tilde{y},$$

which yields

$$\tilde{x}(p_{A_1}, p_{A_2}, p_{B_1}, p_{B_2}) = \frac{1}{2}\big(1 + q_A - q_B + p_{B_1} - p_{A_1} + p_{B_2} - p_{A_2}\big).$$

Profit of firm A is $\Pi_A(p_{A_1}, p_{A_2}, p_{B_1}, p_{B_2}) = p_{A_1}\tilde{x}(p_{A_1}, p_{A_2}, p_{B_1}, p_{B_2}) + p_{A_2}\tilde{y}(p_{A_2}, p_{B_2})$, since at this stage the cost of investment is sunk and I can omit it. Similarly, the profit of firm B is $\Pi_B(p_{A_1}, p_{A_2}, p_{B_1}, p_{B_2}) = p_{B_1}(1 - \tilde{x}(p_{A_1}, p_{A_2}, p_{B_1}, p_{B_2})) + p_{B_2}(\rho - \tilde{y}(p_{A_2}, p_{B_2}))$. The first-order conditions for firm A are:

$$\frac{\partial \Pi_A}{\partial p_{A_1}} = 0 \Leftrightarrow \frac{1}{2}\big(1 + q_A - q_B + p_{B_1} - p_{A_1} + p_{B_2} - p_{A_2}\big) - \frac{1}{2}p_{A_1} = 0, \tag{21}$$

$$\frac{\partial \Pi_A}{\partial p_{A_2}} = 0 \Leftrightarrow p_{A_1}(-\frac{1}{2}) + \frac{\rho}{2} + \frac{1}{2}(p_{B_2} - p_{A_2}) + p_{A_2}(-\frac{1}{2}) = 0. \tag{22}$$

The second-order derivatives for firm A are:

$$\frac{\partial^2 \Pi_A}{\partial p_{A_1}^2} = -1, \tag{23}$$

$$\frac{\partial^2 \Pi_A}{\partial p_{A_2}^2} = -1, \tag{24}$$

$$\frac{\partial \Pi_A}{\partial p_{A_2} \partial p_{A_1}} = -\frac{1}{2}. \tag{25}$$

The Hessian matrix of firm A is therefore

$$H_A = \begin{pmatrix} -1 & -\frac{1}{2} \\ -\frac{1}{2} & -1 \end{pmatrix}.$$

The leading principal minor is $\Delta_1 = -1 < 0$, and the determinant is

$$\Delta_2 = (-1)(-1) - \left(-\tfrac{1}{2}\right)^2 = \tfrac{3}{4} > 0.$$

Hence, H_A is negative definite, so the second-order condition is satisfied. Since firms are symmetric, I can use symmetry to pin down the first-order conditions for firm B and I then get the following equilibrium prices:

$$p_{A_1}(q_A, q_B) = 1 + \frac{3}{7}(q_A - q_B), \qquad p_{B_1}(q_A, q_B) = 1 - \frac{3}{7}(q_A - q_B), \tag{26}$$

$$p_{A_2}(q_A, q_B) = \rho - 1 - \frac{1}{7}(q_A - q_B), \qquad p_{B_2}(q_A, q_B) = \rho - 1 + \frac{1}{7}(q_A - q_B). \tag{27}$$

Substituting the equilibrium prices into the profit function of firm A, and now incorporating the sunk investment cost $C(q_i) = q_i^2/(2\eta)$, yields:

$$\pi_A(q_A, q_B) = \frac{1}{98}(3q_A - 3q_B + 7)^2 + \frac{1}{98}(7\rho + 2q_A - 2q_B)(7\rho - q_A + q_B - 7) - \frac{q_A^2}{2\eta}.$$

The first-order conditions are:

$$\frac{\partial \pi_A}{\partial q_A} = 0 \Leftrightarrow 2q_A - 2q_B + \rho + 4 - \frac{14 q_A}{\eta} = 0, \tag{28}$$

$$\frac{\partial \pi_B}{\partial q_B} = 0 \Leftrightarrow 2q_B - 2q_A + \rho + 4 - \frac{14 q_B}{\eta} = 0. \tag{29}$$

The second-order derivatives are

$$\frac{\partial^2 \pi_A}{\partial q_A^2} = \frac{1}{7} - \frac{1}{\eta}, \tag{30}$$

$$\frac{\partial^2 \pi_B}{\partial q_B^2} = \frac{1}{7} - \frac{1}{\eta}. \tag{31}$$

The second-order condition is satisfied whenever $\eta \leq 7$, which I assume holds throughout. Solving the first-order conditions yields the symmetric equilibrium quality investments

$$q_A^* = q_B^* = \frac{(\rho + 4)\,\eta}{14}.$$

Substituting equilibrium quality investments into equilibrium prices gives

$$p_{A_1}^* = p_{B_1}^* = 1, \qquad p_{A_2}^* = p_{B_2}^* = \rho - 1.$$

Equilibrium quantities are symmetric in both markets,

$$\tilde{x}^* = \frac{1}{2}, \quad \text{and} \quad \tilde{y}^* = \frac{\rho}{2}.$$

and equilibrium profits are

$$\pi_A^* = \pi_B^* = \frac{1}{2}(\rho^2 + 1 - \rho) - \frac{\eta\,(\rho + 4)^2}{392}.$$

And by similar reasonning to the benchmark case, the consumer surplus is

$$CS^* = v_1 + \frac{1}{14}\eta(\rho + 4) + \frac{1}{4}(\rho(-5\rho + 4v_2 + 6) - 5).$$

□

Proof of Proposition 2. I compare the symmetric equilibrium outcomes of the baseline model with those of the benchmark for each sub-proposition.

1. Proof is obvious for equilibrium prices.

2. Comparing equilibrium profits,

$$\pi_i^* = \frac{1}{2}\left(\rho^2 - \rho + 1\right) - \frac{\eta(\rho + 4)^2}{392}, \qquad \pi_i^b = \frac{9 - \eta}{18},$$

   I obtain, over a common denominator,

$$\pi_i^* - \pi_i^b = \frac{(1764 - 9\eta)\,\rho^2 - (1764 + 72\eta)\,\rho + 52\eta}{3528}.$$

Denote the numerator by $g(\rho)$. Since $\eta < 7$, its leading coefficient $1764 - 9\eta > 0$, so g is a convex parabola. Evaluating at the endpoints, I have

$$g(0) = 52\eta > 0, \qquad \text{and} \qquad g(1) = -29\eta < 0.$$

Since $g(0) > 0$ and $g(1) < 0$, the Intermediate Value Theorem implies that g has a root in $(0, 1)$. Moreover, g is convex because $1764 - 9\eta > 0$. Since $g(1) < 0$, the vertex of g must lie to the right of 1, so the second root is greater than 1. Hence g has exactly one root in $(0, 1)$, namely the smaller of its two roots, and $g(\rho) > 0$ for all ρ below it. Solving $g(\rho) = 0$ gives

$$\rho_\pi = \frac{2\big(6\eta + 147 - 7\sqrt{\eta^2 - 16\eta + 441}\big)}{3\,(196 - \eta)},$$

where the radicand $\eta^2 - 16\eta + 441 > 0$ for all η. Hence

$$\pi_i^* > \pi_i^b \iff \rho < \rho_\pi,$$

and profits are lower than in the benchmark otherwise.

3. Consumer surplus in the benchmark is

$$CS^b = v_1 + \frac{\eta}{3} - \frac{5}{4},$$

while in the baseline model,

$$CS^* = v_1 + \frac{\eta(\rho + 4)}{14} + \frac{1}{4}\big(\rho(-5\rho + 4v_2 + 6) - 5\big).$$

Their difference is

$$CS^* - CS^b = \frac{-105\,\rho^2 + (6\eta + 84v_2 + 126)\,\rho - 4\eta}{84}.$$

Denote the numerator by $h(\rho)$. Its leading coefficient is negative, so h is a concave parabola, and at the endpoints

$$h(0) = -4\eta < 0, \qquad h(1) = 21 + 2\eta + 84v_2 > 0,$$

so, by similar argument as in the proof for profits, h has exactly one root in $(0, 1)$, the smaller of its two roots, with $h(\rho) < 0$ for ρ below it. Solving $h(\rho) = 0$ gives

$$\rho_{CS} = \frac{1}{105}\left[3\eta + 42v_2 + 63 \;-\; \sqrt{(3\eta + 42v_2 + 63)^2 - 420\,\eta}\right].$$

Hence

$$CS^* < CS^b \iff \rho < \rho_{CS},$$

and consumer surplus is higher than in the benchmark otherwise.

□

Computations for the equilibrium with secondary-market transport cost. I now suppose that in the secondary market, the consumers incur a transportation cost $t \geq 1$ per unit of distance, where $t = 1$ is our baseline. By backward induction, the indifferent secondary-market consumer now satisfies

$$v_2 - p_{A_2} - t\tilde{y} = v_2 - p_{B_2} - t(\rho - \tilde{y}),$$

which yields $\tilde{y} = \frac{\rho}{2} + \frac{1}{2t}(p_{B_2} - p_{A_2})$. Going through the same process as in the baseline model—solving for the consumer demands, then the primary- and secondary-market prices, substituting into profits, and solving the quality stage with investment cost $C(q_i) = q_i^2/(2\eta)$—the quality-stage second-order condition is

$$\frac{\partial^2 \pi_i}{\partial q_i^2} = \frac{t}{9t-2} - \frac{1}{\eta} < 0 \quad \iff \quad \eta < 9 - \frac{2}{t},$$

which holds given our previous assumption that $\eta \leq 7$. I obtain the symmetric equilibrium outcomes

$$p^t_{A_1} = p^t_{B_1} = 1, \qquad p^t_{A_2} = p^t_{B_2} = t\rho - 1,$$
$$q^t_A = q^t_B = \frac{\eta\,(t\rho + 6t - 2)}{2(9t-2)}, \qquad x^t = \frac{1}{2}, \qquad y^t = \frac{\rho}{2},$$

and equilibrium profits

$$\pi^t_A = \pi^t_B = \frac{t\rho^2 - \rho + 1}{2} - \frac{\eta\,(t\rho + 6t - 2)^2}{8(9t-2)^2}.$$

Total consumer surplus is

$$CS^t = v_1 + v_2\rho + \frac{6\rho - 5t\rho^2 - 5}{4} + \frac{\eta\,(t\rho + 6t - 2)}{2(9t-2)}.$$

□

Proof of Proposition 3. Write $D := 9t - 2 > 0$. The benchmark profit is $\pi_i^b = \frac{9-\eta}{18}$ and the transport-cost profit is

$$\pi_i^t = \frac{t\rho^2 - \rho + 1}{2} - \frac{\eta\,(t\rho + 6t - 2)^2}{8D^2}.$$

The gap is a quadratic in ρ. Placing $\pi_i^t - \pi_i^b$ over the common denominator $72D^2 > 0$ gives $\pi_i^t - \pi_i^b = \dfrac{G(\rho)}{72D^2}$, where

$$G(\rho) := 36D^2\big(t\rho^2 - \rho + 1\big) - 9\eta\,(t\rho + 6t - 2)^2 - 4D^2(9 - \eta).$$

Using $(t\rho+6t-2)^2 = t^2\rho^2+4t(3t-1)\rho+4(3t-1)^2$ and the identity $D^2-9(3t-1)^2 = 18t-5$, I collect powers of ρ:

$$G(\rho) = A\rho^2 + B\rho + C,$$

$$A = 9t\big(4D^2 - \eta t\big), \qquad B = -36\big[D^2 + \eta t(3t - 1)\big], \qquad C = 4\eta(18t - 5).$$

Thus $\operatorname{sign}\big(\pi_i^t - \pi_i^b\big) = \operatorname{sign} G(\rho)$.

G is convex. The sign of the expression A simplifies to the sign of $4D^2/t - \eta$. I find that $4D^2/t = 4\big(81t - 36 + 4/t\big)$ is increasing in t on $[1, \infty)$ with minimum value 196 at $t = 1$, so $4D^2/t - \eta > 0$ for all $\eta \le 7$, $t \ge 1$. Hence $A = 9t(4D^2 - \eta t) > 0$ and G is strictly convex.

The larger root. The discriminant is $B^2 - 4AC$. Expanding this discriminant, with $u := \eta t$,

$$B^2 - 4AC = 144\Big(9\big[D^2 + u(3t - 1)\big]^2 - u(4D^2 - u)(18t - 5)\Big) = 144\,D^2 R,$$

where, using $9(3t - 1)^2 + (18t - 5) = D^2$, and

$$R := 9D^2 - 2(9t - 1)\,\eta t + \eta^2 t^2 = 9(9t - 2)^2 - 2\eta t(9t - 1) + \eta^2 t^2.$$

As a quadratic in η, R has discriminant $4t^2\big[(9t-1)^2-9(9t-2)^2\big] = 4t^2\big(-648t^2+306t-35\big)$, and $-648t^2 + 306t - 35$ equals -377 at $t = 1$ and is decreasing for $t \ge 1$, hence the discriminant (of R) is negative. Since the discriminant of R is negative and the leading coefficient of R is positive, I have $R > 0$ for all admissible (t, η). As a result, the discriminant of G is positive and G has two distinct real roots. The larger root is

$$\rho_+ = \frac{-B + \sqrt{B^2 - 4AC}}{2A} = \underline{\rho}(t, \eta),$$

which is exactly the expression in the statement after substitution.

Since $A > 0$, I have $G(\rho) > 0$ for every $\rho > \underline{\rho}$. Hence for all $\rho \in (\underline{\rho}(t,\eta),\, 1]$,

$$\pi_i^t - \pi_i^b = \frac{G(\rho)}{72(9t-2)^2} > 0,$$

i.e. firms' equilibrium profits strictly exceed the benchmark level. □

Proof of Proposition 4. Fix $\eta \in [1,7]$ and write $D := 9t - 2 \geq 7$. All quantities below are evaluated at the symmetric transport-cost equilibrium and compared to the benchmark.

- *Profits.* For convenience, I will use some notations and results in the proof of Proposition 3. By Proposition 3, I have

$$\pi_i^t > \pi_i^b \quad \text{for all } \rho > \underline{\rho}(t,\eta).$$

This region is non-empty iff $\underline{\rho}(t,\eta) < 1$, or equivalently iff $\pi_i^t - \pi_i^b = G(1)/(72D^2) > 0$ which reduces to $G(1) > 0$; since $G(1)\big|_{t=1} = -29\eta < 0$ while $G(1)\big|_{t=2} = 9216 - 272\eta > 0$ for all $\eta \leq 7$, the region is non-empty for every t large enough (in particular for t in a neighbourhood of 2). Fix such a t.

Consumer Surplus gap and its threshold. Subtracting $CS^b = v_1 + \frac{\eta}{3} - \frac{5}{4}$ from CS^t and using $\frac{t\rho+6t-2}{2(9t-2)} - \frac{1}{3} = \frac{3t\rho-2}{6(9t-2)}$,

$$\Delta_{CS}(\rho) := CS^t - CS^b = -\frac{5t}{4}\rho^2 + \left(v_2 + \tfrac{3}{2} + \tfrac{\eta t}{2(9t-2)}\right)\rho - \frac{\eta}{3(9t-2)}.$$

This is strictly concave in ρ, with $\Delta_{CS}(0) = -\frac{\eta}{3(9t-2)} < 0$. Hence, provided

$$\Delta_{CS}(1) = v_2 + \frac{6-5t}{4} + \frac{\eta(3t-2)}{6(9t-2)} > 0 \qquad (*)$$

which holds for the maintained levels of v_2 ensuring secondary-market participation (e.g. $v_2 \geq 1$ suffices at $t = 2$), Δ_{CS} has exactly one root $\rho_{CS} \in (0,1)$ and its other root exceeds 1, so

$$CS^t > CS^b \quad \Longleftrightarrow \quad \rho > \rho_{CS} \qquad \text{on } (0,1].$$

The profit threshold is the binding one: $\rho_{CS} < \underline{\rho}$. I bound the two thresholds.

Upper bound on ρ_{CS}. The roots of the concave Δ_{CS} are both positive (positive sum and product), and their product equals $\frac{4\eta}{15t(9t-2)}$. Under $(*)$ the larger root exceeds 1, so

$$\rho_{CS} < \frac{4\eta}{15t(9t-2)}.$$

Lower bound on $\underline{\rho}$. Being the larger root of the convex G, $\underline{\rho}$ is at least the half-sum of the roots,

$$\underline{\rho} \;\geq\; \frac{2\left[D^2 + \eta t(3t-1)\right]}{t\,(4D^2 - \eta t)} \;>\; \frac{2D^2}{t \cdot 4D^2} = \frac{1}{2t},$$

using $\eta t(3t-1) > 0$ in the numerator and $4D^2 - \eta t < 4D^2$ in the denominator.

Comparison. For all $t \geq 1$ and $\eta \leq 7$,

$$\frac{4\eta}{15t(9t-2)} < \frac{1}{2t} \quad \Longleftrightarrow \quad 8\eta < 15(9t-2) = 135t - 30,$$

and the right-hand inequality holds since $135t - 30 \geq 105 > 56 \geq 8\eta$. Chaining the three implies,

$$\rho_{CS} < \frac{4\eta}{15t(9t-2)} < \frac{1}{2t} < \underline{\rho}.$$

For every $\rho \in (\underline{\rho}(t,\eta),\, 1]$ I have $\rho > \underline{\rho} > \rho_{CS}$, and hence I find both

$$\pi_i^t > \pi_i^b \qquad \text{and} \qquad CS^t > CS^b,$$

□

Computations for unilateral entry equilibrium outcomes (verify deviation..). Suppose that only firm A expands the secondary market. Firm A incurs the fixed entry cost f, while firm B remains active only in the primary market. Because firm B does not operate in the secondary market, firm A can capture the whole secondary market by setting a price such that all consumers strictly prefer buying from A. The marginal secondary-market consumer satisfies

$$v_2 - p_{A_2} - y = 0,$$

so demand for firm A in the secondary market is

$$y_A = v_2 - p_{A_2}.$$

Since demand cannot exceed the market size ρ, firm A optimally sets the highest price compatible with full market coverage:

$$\rho = v_2 - p_{A_2},$$

which yields

$$p_{A_2} = v_2 - \rho.$$

At this price, firm A captures all secondary-market consumers and obtains

$$d_A = \rho, \qquad d_B = 0.$$

In the primary market, the indifferent consumer satisfies

$$q_A + d_A - p_{A_1} - x = q_B + d_B - p_{B_1} - (1 - x).$$

Substituting $d_A = \rho$ and $d_B = 0$ gives

$$\tilde{x}^{E_A}(p_{A_1}, p_{B_1}) = \frac{1}{2} + \frac{1}{2}(q_A - q_B + \rho + p_{B_1} - p_{A_1}).$$

Operating profits are

$$\Pi_A = p_{A_1}\tilde{x}^{E_A} + \rho(v_2 - \rho),$$

and

$$\Pi_B = p_{B_1}(1 - \tilde{x}^{E_A})$$

The first-order conditions with respect to primary-market prices are

$$\frac{\partial \Pi_A}{\partial p_{A_1}} = 0 \Leftrightarrow \frac{1}{2} + \frac{1}{2}(q_A - q_B + \rho + p_{B_1} - p_{A_1}) + (-\frac{1}{2})p_{A_1} = 0, \qquad (32)$$

$$\frac{\partial \Pi_B}{\partial p_{B_1}} = 0 \Leftrightarrow \frac{1}{2} - \frac{1}{2}(q_A - q_B + \rho + p_{B_1} - p_{A_1}) + (-\frac{1}{2})p_{B_1} = 0. \qquad (33)$$

The second-order derivatives are

$$\frac{\partial^2 \Pi_A}{\partial p_{A_1}^2} = -1, \qquad \frac{\partial^2 \Pi_B}{\partial p_{B_1}^2} = -1,$$

so the second-order condition is satisfied. Solving the first-order conditions yields equilibrium prices

$$p_{A_1}^{E_A} = 1 + \frac{1}{3}(q_A - q_B + \rho),$$

and

$$p_{B_1}^{E_A} = 1 - \frac{1}{3}(q_A - q_B + \rho).$$

Substituting equilibrium prices into profits gives

$$\pi_A(q_A, q_B) = \frac{(q_A - q_B + \rho + 3)^2}{18} + \rho(v_2 - \rho) - \frac{q_A^2}{2\eta} - f,$$

and

$$\pi_B(q_A, q_B) = \frac{(3 - q_A + q_B - \rho)^2}{18} - \frac{q_B^2}{2\eta}.$$

The first-order conditions with respect to quality investments are

$$\frac{\partial \pi_A}{\partial q_A} = 0 \Leftrightarrow \frac{q_A - q_B + \rho + 3}{9} - q_A/\eta = 0, \tag{34}$$

$$\frac{\partial \pi_B}{\partial q_B} = 0 \Leftrightarrow \frac{-q_A + q_B - \rho + 3}{9} - q_B/\eta = 0. \tag{35}$$

The second-order derivatives are

$$\frac{\partial^2 \pi_A}{\partial q_A^2} = \frac{1}{9} - \frac{1}{\eta}, \qquad \frac{\partial^2 \pi_B}{\partial q_B^2} = \frac{1}{9} - \frac{1}{\eta},$$

so the second-order condition is satisfied since I assumed $\eta \leq 7$. Solving the system yields equilibrium quality investments

$$q_A^{E_A} = \eta \left(\frac{\rho}{9 - 2\eta} + \frac{1}{3} \right), \qquad q_B^{E_A} = \eta \left(\frac{1}{3} - \frac{\rho}{9 - 2\eta} \right).$$

Substituting equilibrium qualities into equilibrium prices gives

$$p_{A_1}^{E_A} = 1 + \frac{3\rho}{9 - 2\eta}, \qquad p_{B_1}^{E_A} = 1 - \frac{3\rho}{9 - 2\eta},$$

and

$$p_{A_2}^{E_A} = v_2 - \rho.$$

Equilibrium demand in the primary market is

$$x^{E_A} = \frac{1}{2} + \frac{3\rho}{18 - 4\eta}.$$

Finally, equilibrium profits are

$$\pi_A^{E_A} = \frac{(9 - \eta)\big(9 - 2\eta + 3\rho\big)^2}{18(9 - 2\eta)^2} + v_2\rho - \rho^2 \quad \text{and} \quad \pi_B^{E_A} = \frac{(9 - \eta)\big(9 - 2\eta - 3\rho\big)^2}{18(9 - 2\eta)^2}.$$

It remains to verify that firm A does not prefer to raise p_{A_2} above $v_2 - \rho$ and serve less than the full secondary market.

To do that, I suppose firm A deviates and sets

$$p'_{A_2} = v_2 - \rho + \epsilon, \qquad \epsilon > 0.$$

Demand in the secondary market becomes

$$d_A = \rho - \epsilon, \qquad d_B = 0.$$

Given qualities $q_A^{E_A}$ and $q_B^{E_A}$, the indifferent consumer in the primary market is

$$\tilde{x}(\epsilon) = \frac{1}{2} + \frac{1}{2}\Big(q_A^{E_A} - q_B^{E_A} + \rho - \epsilon + p_{B_1} - p_{A_1}\Big).$$

The resulting equilibrium primary-market prices are therefore

$$p_{A_1}(\epsilon) = 1 + \frac{1}{3}\Big(q_A^{E_A} - q_B^{E_A} + \rho - \epsilon\Big),$$

and

$$p_{B_1}(\epsilon) = 1 - \frac{1}{3}\Big(q_A^{E_A} - q_B^{E_A} + \rho - \epsilon\Big).$$

Substituting these prices into firm A's primary-market profit yields

$$\Pi_{A_1}(\epsilon) = \frac{\Big(q_A^{E_A} - q_B^{E_A} + \rho - \epsilon + 3\Big)^2}{18}.$$

Using

$$q_A^{E_A} - q_B^{E_A} = \frac{2\eta\rho}{9 - 2\eta},$$

I obtain

$$\Pi_{A_1}(\epsilon) = \frac{\Big(3 + \frac{9\rho}{9-2\eta} - \epsilon\Big)^2}{18}.$$

Secondary-market profit is

$$\begin{aligned}\Pi_{A_2}(\epsilon) &= (v_2 - \rho + \epsilon)(\rho - \epsilon)\\ &= \rho(v_2 - \rho) + \epsilon(2\rho - v_2) - \epsilon^2.\end{aligned}$$

Hence total profit under the deviation is

$$\pi_A(\epsilon) = \frac{\Big(3 + \frac{9\rho}{9-2\eta} - \epsilon\Big)^2}{18} + (v_2 - \rho + \epsilon)(\rho - \epsilon) - \frac{(q_A^{E_A})^2}{2\eta}.$$

Subtracting equilibrium profit,

$$\pi_A(0) = \frac{\Big(3 + \frac{9\rho}{9-2\eta}\Big)^2}{18} + \rho(v_2 - \rho) - \frac{(q_A^{E_A})^2}{2\eta},$$

gives

$$\pi_A(\epsilon) - \pi_A(0) = \frac{\Big(3 + \frac{9\rho}{9-2\eta} - \epsilon\Big)^2 - \Big(3 + \frac{9\rho}{9-2\eta}\Big)^2}{18} + \epsilon(2\rho - v_2) - \epsilon^2.$$

Using the identity

$$(a-\epsilon)^2 - a^2 = -2a\epsilon + \epsilon^2,$$

with

$$a = 3 + \frac{9\rho}{9-2\eta},$$

I obtain

$$\begin{aligned}\pi_A(\epsilon) - \pi_A(0) &= -\frac{1}{9}\left(3 + \frac{9\rho}{9-2\eta}\right)\epsilon + \frac{\epsilon^2}{18} + \epsilon(2\rho - v_2) - \epsilon^2 \\ &= \epsilon\left[2\rho - v_2 - \frac{1}{3} - \frac{\rho}{9-2\eta}\right] - \frac{17}{18}\epsilon^2.\end{aligned}$$

Since the coefficient of ϵ^2 is negative, the deviation is unprofitable for all sufficiently small $\epsilon > 0$ whenever

$$v_2 \geq 2\rho - \frac{1}{3} - \frac{\rho}{9-2\eta}.$$

Under this condition, firm A prefers to serve the entire secondary market and the candidate equilibrium price

$$p_{A_2}^{E_A} = v_2 - \rho$$

is indeed optimal.

□

Proof of Proposition 5. *Best responses.* Read the payoffs from Table 1. Given that the rival plays N, firm A earns $\pi_A^{E_A} - f$ from expanding and π_A^b from not expanding, so expansion is a best response if and only if $f \leq \pi_A^{E_A} - \pi_A^b = f^{EN}$. Given that the rival plays E, firm A earns $\pi_A^* - f$ from expanding and $\pi_A^{E_B}$ from not expanding, so expansion is a best response if and only if $f \leq \pi_A^* - \pi_A^{E_B} = f^{EE}$. By symmetry the same holds for firm B. Hence (E, E) is a Nash equilibrium if and only if $f \leq f^{EE}$; (N, N) if and only if $f \geq f^{EN}$; and an asymmetric profile, say (E, N), if and only if the expansionist does not deviate ($f \leq f^{EN}$) and the abstainer does not deviate ($f \geq f^{EE}$), i.e. $f^{EE} \leq f \leq f^{EN}$.

Ordering (12). Write

$$f^{EN} - f^{EE} = \pi_A^{E_A} + \pi_A^{E_B} - \pi_A^* - \pi_A^b = \underbrace{\left(\pi_A^{E_A} - \pi_A^*\right)}_{\text{(I)}} + \underbrace{\left(\pi_A^{E_B} - \pi_A^b\right)}_{\text{(II)}}.$$

Term (II) is non-positive: the maintained condition $\eta \leq 3$ implies $2\eta + 3\rho \leq 9$ for every $\rho \leq 1$, hence $0 \leq 9 - 2\eta - 3\rho \leq 9 - 2\eta$ and

$$\frac{\pi_A^{E_B}}{\pi_A^b} = \left(\frac{9-2\eta-3\rho}{9-2\eta}\right)^2 \leq 1.$$

Term (I) is strictly positive and dominates: the only term carrying v_2 is the monopoly rent $+v_2\rho$ in $\pi_A^{E_A}$, so $\partial(f^{EN} - f^{EE})/\partial v_2 = \rho > 0$; the difference is therefore increasing in v_2 and strictly positive at the maintained $v_2 = 2$ for every $\rho \geq \underline{\rho}_0$, the restriction adopted in the text (only on the negligible interval $(0, \underline{\rho}_0)$, where the secondary market vanishes, does the ordering reverse). Finally $f^{EE} > 0$, i.e. $\pi_A^* > \pi_A^{E_B}$, holds on this range, so case 1 is non-empty.

Equilibrium structure. Because $f^{EE} < f^{EN}$, the three best-response conditions partition the cost range. If $f < f^{EE}$, only the first applies, giving the unique equilibrium (E, E): the asymmetric profiles fail since $f < f^{EE}$, and (N, N) fails since $f < f^{EE} < f^{EN}$. If $f^{EE} < f < f^{EN}$, only the asymmetric condition applies, giving (E, N) and (N, E); neither (E, E) (as $f > f^{EE}$) nor (N, N) (as $f < f^{EN}$) survives. If $f > f^{EN}$, only the third applies, giving the unique equilibrium (N, N). (At the boundaries $f = f^{EE}$ and $f = f^{EN}$ the adjacent profiles coincide.)

Prisoner's dilemma. In case 1 the equilibrium payoff is $\pi_A^* - f$, which lies below the mutual-abstention payoff π_A^b if and only if $f > \pi_A^* - \pi_A^b$. At $f = 0$ this reduces to $\pi_A^* < \pi_A^b$; by the algebra of Proposition 2, where $\pi_A^* = \pi_A^b$ defines ρ_π and $\pi_A^* < \pi_A^b \iff \rho > \rho_\pi$, mutual expansion is Pareto-dominated by mutual abstention exactly on $\rho > \rho_\pi$. □

Computations for the expansion game with secondary-market transport cost. Reintroduce the transport cost $t \geq 1$ of Section 5 in the secondary market. Mutual expansion then yields the symmetric profit derived above,

$$\pi_i^t = \frac{t\rho^2 - \rho + 1}{2} - \frac{\eta\,(t\rho + 6t - 2)^2}{8(9t - 2)^2},$$

while the abstainer of the unilateral-expansion subgame is active only in the primary market and therefore earns the same profit as under $t = 1$,

$$\pi_A^{E_B,t} = \pi_A^{E_B} = \frac{(9 - \eta)(9 - 2\eta - 3\rho)^2}{18(9 - 2\eta)^2},$$

independent of t. The sole expansionist covers the whole secondary segment at $p_{A_2}^{E_A} = v_2 - t\rho$ (optimal when $v_2 \geq 2t\rho$), so its data are still $d_A = \rho$ and the primary stage is unchanged; only its secondary revenue becomes $(v_2 - t\rho)\rho$, giving

$$\pi_A^{E_A,t} = \frac{(9 - \eta)(9 - 2\eta + 3\rho)^2}{18(9 - 2\eta)^2} + v_2\rho - t\rho^2.$$

The expansion thresholds are $f^{EN,t} = \pi_A^{E_A,t} - \pi_A^b$ and $f^{EE,t} = \pi_i^t - \pi_A^{E_B}$, and Proposition 5 applies verbatim with these thresholds.

Both firms gain from endogenous expansion. At $f = 0$ the unique equilibrium is (E, E) whenever $f^{EE,t} > 0$ and $f^{EN,t} > 0$, and both firms then strictly beat the benchmark whenever $\pi_i^t > \pi_A^b$. Both conditions on the equilibrium being (E, E) and on the benchmark comparison involve only π_i^t, $\pi_A^{E_B}$ and π_A^b; they are therefore independent of the entrant's coverage decision. By Proposition 3, $\pi_i^t > \pi_A^b$ for every $\rho > \underline{\rho}(t, \eta)$, and on that range one checks that $\pi_i^t > \pi_A^{E_B}$ (so $f^{EE,t} > 0$) and $f^{EN,t} > 0$. Hence there is a non-empty set of parameters—a sufficiently large and sufficiently differentiated secondary market—on which both firms expand in equilibrium and each earns strictly more than under no expansion. This region is exactly the large-ρ range that, at $t = 1$, constitutes the prisoner's dilemma of Proposition 5: differentiation converts that dilemma into a mutually beneficial outcome. For instance, at $\eta = 2$, $v_2 = 2$ and $t = 1.3$ this holds for all $\rho \in (0.80, 1]$, whereas the same configuration is a prisoner's dilemma for all $\rho > 0.06$ when $t = 1$. □

Note: In addition to the following proofs of the robustness checks, a Mathematica file is also available to replicate the results.

Proof for the R&D spillover on secondary market. Replace the secondary value v_2 by $v_2 + s\, q_i$, $s \in [0, 1]$, and keep a secondary transport cost $t \geq 1$ (the baseline is $t = 1$). The indifferent consumers become

$$\tilde{y} = \frac{\rho}{2} + \frac{1}{2t}\big(s(q_A - q_B) + p_{B_2} - p_{A_2}\big), \qquad \tilde{x} = \tfrac{1}{2}\big(1 + q_A - q_B + (2\tilde{y} - \rho) + p_{B_1} - p_{A_1}\big).$$

Solving the pricing stage and then the symmetric quality stage (the Hessian is unchanged at the pricing stage; the quality second-order condition is $\frac{1}{\eta} > \frac{1+s+s^2}{7}$ at $t = 1$, i.e. $\eta < \frac{7}{1+s+s^2}$), the symmetric equilibrium is

$$p_1^* = 1, \qquad p_2^* = t\rho - 1, \qquad q^* = \frac{\eta\big[(6+\rho)t - 2 + s\big(\rho(6t-1) - 1\big)\big]}{2(9t-2)},$$

which collapses to the baseline ($q^* = \frac{(4+\rho)\eta}{14}$ at $t = 1$, $s = 0$). Two comparative statics follow immediately. First, $\partial q^*/\partial\rho = \frac{\eta(1+5s)}{14}$ at $t = 1$, strictly larger than the baseline $\frac{\eta}{14}$: equilibrium quality is steeper in ρ. Second, $q^* = q^b = \eta/3$ at $\rho_s = \frac{2+3s}{3(1+5s)}$, decreasing in s from $\frac{2}{3}$ at $s = 0$; firms invest less than the benchmark for $\rho < \rho_s$ and more for $\rho > \rho_s$. Hence the conclusion of Proposition 1 survives, with a lower threshold.

Proposition 2 survives ($t = 1$). Writing the profit gap as $\pi_i^* - \pi_i^b = g(\rho)/3528$, the numerator is a quadratic in ρ with

$$g(0) = \eta\big(52 + 72s - 9s^2\big) > 0, \qquad g(1) = -\eta\big(29 + 360s + 144s^2\big) < 0 \quad \text{for } s \in [0, 1],$$

so g has exactly one root in $(0,1)$ and $\pi_i^* > \pi_i^b \iff \rho < \rho_\pi(s)$. Likewise the consumer-surplus gap $CS^* - CS^b = h(\rho)/84$ has

$$h(0) = -\eta(4+6s) < 0, \qquad h(1) = 21 + 84v_2 + \eta(2 + 54s + 24s^2) > 0,$$

so $CS^* < CS^b \iff \rho < \rho_{CS}(s)$. Both single-crossing properties of Proposition 2 therefore hold, at thresholds that shift with s.

Proposition 4 survives ($t > 1$). By the same construction with $t > 1$, the secondary price $p_2^* = t\rho - 1$ and the profit and surplus gaps retain the same endpoint signs, so the larger root of the (convex) profit gap and the unique surplus root again order as $\rho_{CS} < \underline{\rho}$. The set of (ρ, t) on which both $\pi_i^* > \pi_i^b$ and $CS^* > CS^b$ is non-empty (verified for, e.g., $s = \frac{1}{2}$, $t \in \{1.2, 1.3\}$, $\eta = 1$, $v_2 = 2$). The Pareto-improvement region of Proposition 4 thus persists. □

Proof for the reduced data-driven channel δ. Scale the data term by $\delta \in [0,1]$, so primary utility is $v_1 + q_i + \delta d_i - p_{i_1} - x$, and keep $t \geq 1$. The indifferent secondary consumer is unchanged, while $\tilde{x} = \frac{1}{2}\big(1 + q_A - q_B + \delta(2\tilde{y} - \rho) + p_{B_1} - p_{A_1}\big)$. Solving the pricing and quality stages gives the symmetric equilibrium

$$p_1^* = 1, \qquad p_2^* = t\rho - \delta, \qquad q^* = \frac{\eta\big(\delta\rho t + 6t - 2\delta^2\big)}{2\big(9t - 2\delta^2\big)},$$

with quality second-order condition $\frac{1}{\eta} > \frac{t}{9t - 2\delta^2}$, i.e. $\eta < 9 - \frac{2\delta^2}{t}$. All outcomes vary continuously in δ and reduce to the transport-cost model at $\delta = 1$ (and to the baseline at $\delta = 1$, $t = 1$). The equilibrium is therefore symmetric, interior, and continuous in δ.

Limit $\delta = 0$. Here $q^* = \eta/3 = q^b$, $p_2^* = t\rho > 0$, and

$$\pi_i^* = \pi_i^b + \tfrac{1}{2}t\rho^2 > \pi_i^b \quad \text{for all } \rho \in (0,1].$$

The data channel vanishes: quality reverts to the benchmark, the secondary market is simply an additional profitable segment, and profits exceed the benchmark for every market size.

Proposition 2 for $\delta \in (0,1]$ ($t = 1$). The profit gap $\pi_i^* - \pi_i^b$ is a quadratic in ρ with

$$g(0) = 4\delta^2\eta\big(18 - 5\delta^2\big) > 0 \quad \text{for all } \delta \in (0,1],$$

so profits exceed the benchmark on a neighbourhood of $\rho = 0$ for every $\delta > 0$. The endpoint $g(1)$ is strictly decreasing in δ and changes sign at some $\delta^*(\eta)$:

- if $\delta > \delta^*(\eta)$, then $g(1) < 0$ and g single-crosses in $(0, 1)$, so $\pi_i^* > \pi_i^b \iff \rho < \rho_\pi(\delta)$, exactly as in Proposition 2;
- if $\delta < \delta^*(\eta)$, then $g(1) > 0$ while $g(0) > 0$ and g is convex, so the small-market region $\pi_i^* > \pi_i^b$ is joined by an additional large-market region, and the "if and only if" is replaced by a union of two intervals.

Proposition 4 for $\delta \in (0, 1]$. At $\delta = 1$ the model coincides with the transport-cost model, so by Proposition 4 there exist interior (ρ_0, t_0) at which $\pi_i^* - \pi_i^b > 0$ and $CS^* - CS^b > 0$ strictly. Both gaps are functions of δ on the admissible set $\eta < 9 - 2\delta^2/t$ and hence continuous in δ; the strict inequalities therefore persist on a neighbourhood of $\delta = 1$. The set of (ρ, t) on which firms and consumers are simultaneously above the benchmark is thus non-empty for δ close to 1, and remains so well below it (verified for $\delta \in \{0.7, 0.4, 0.2\}$, $t \in \{1.2, 1.5, 2\}$, $\eta \leq 7$, $v_2 = 2$). The Pareto-improvement region of Proposition 4 is therefore robust to a weaker data-driven channel. □