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Optimal electricity consumption and storage under short-term renewable supply variability*

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Abstract

The expansion of intermittent electricity increases supply variability and requires greater flexibility from consumers. This results in welfare losses for these agents, which can nevertheless be mitigated by energy storage. Our model analyzes these welfare consequences in the context of short-term variability in renewable energy given fixed dispatchable and storage capacities. We explore an optimal control problem that determines a welfare-maximizing electricity consumption path by adjusting dispatchable and stored energy throughout the short-term production cycle of renewables. This optimization problem identifies three regimes (no storage and active storage, with or without capacity constraints) and provides the associated consumer welfare over this cycle. Under all three regimes, a certain degree of consumer flexibility is part of the optimal solution and entails welfare losses. Active storage reduces these losses but cannot eliminate them completely due to the energy conversion losses induced by this activity. However, when storage capacity is constrained, a proactive adjustment of this capacity can offset the losses.

Keywords: intermittent renewable, energy storage, electricity consumption, welfare analysis, optimal control

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1. Introduction

The ambition of achieving net-zero greenhouse gas emissions by 2050, as pledged by the European Union and other global actors, has accelerated the deployment of renewable energy sources (IEA 2024). Yet, the intermittency of these sources, particularly in power generation, leads to variability in energy supply. Such imbalances between supply and demand are often managed through flexible generation, typically provided by dispatchable fossil-fueled plants (IEA 2020), though this reliance directly conflicts with decarbonization goals. Another way to maintain electricity balance is on the demand side through dynamic pricing mechanisms such as real-time pricing, which encourage consumers to adjust their consumption in response to supply variability (IRENA 2019). However, this raises a fundamental question: Should the burden of adjusting to supply variability rest primarily on consumers?

Alongside demand-side adjustments, energy storage can play a key role in smoothing the variability of renewable generation, thereby reducing the pressure on consumers to respond to such fluctuations (Jiang et al. 2014, Denholm 2015, Lund et al. 2015, Maghami et al. 2024). This role is particularly relevant in a context where consumers are increasingly expected to adjust their consumption patterns instantaneously to short-term variations in supply (Cabot and Villavicencio 2024). Building on this, the present work starts from the premise that storage of surplus renewable energy can act as an alternative to dispatchable generation and limit the need for short-term demand-side adjustments. Hence, our analysis adopts a short-term perspective, providing a framework to analyze flexibility and efficiency in balancing supply and demand under renewable variability.

Within this short-term focus, we examine a single production cycle driven by intermittent renewables. We therefore abstract from long-term energy transition issues, such as investment and capacity expansion decisions aimed at overcoming renewable intermittency, as explored by Ambec and Crampes (2019), Helm and Mier (2021), and Pommeret and Schubert (2022). Instead, we conduct our analysis within a fixed-capacity framework to study how storage and demand-side adjustments jointly mitigate supply variability at each instant of the cycle. In this context, flexibility is understood as the coordination of consumption with electricity generation and storage facilities, rather than shifts in demand in response to price signals.[1] As in Pommeret and Schubert (2022), we adopt the perspective of a social planner but rather take a short-term view and study the optimal allocation of electricity for consumption under fixed generation and storage capacities. This approach allows us to value storage in terms of consumer welfare rather than solely for its role in cost-minimizing generation, which has been the primary focus of the broader literature (e.g., Crampes and Moreaux 2010, Carson and Novan 2013, Steffen and Weber 2013, Sioshansi 2014, Zerrahn et al. 2018).

Our main objective is to study how different sources of electricity are valued by con-

[1] For an overview of the literature on demand-side response to price signals, see, for instance, Borenstein 2005, Borenstein and Holland, 2005, Joskow and Wolfram 2012, Puller and West 2013, Ambec and Crampes 2021, and Schittekatte et al. 2024.

sumers and to highlight the welfare effects attributable to energy storage. In particular, we find that substituting dispatchable energy with renewables–while maintaining the total energy supply unchanged–reduces consumer welfare. This reduction arises because the variability of renewable generation translates into greater fluctuations in the optimal consumption path, which consumers must accommodate. However, active storage mitigates this effect by allowing surplus energy to be stored and dispatched when needed, thereby smoothing consumption and reducing welfare losses. Furthermore, when storage capacity is binding, a moderate expansion can fully offset the negative welfare effect, underscoring its role in supporting the integration of intermittent renewables from a consumer welfare perspective.

We formalize these results through the resolution of a theoretical model in which the social planner faces a constrained intertemporal allocation problem: how to distribute limited energy resources–dispatchable generation, intermittent renewable production, and stored energy subject to conversion losses–to maximize consumer utility over a single production cycle. This cycle represents a repetitive pattern of renewable generation with a peak around which output fluctuates, reflecting the intermittency of resources such as wind or sunlight (see Helm and Mier 2021). Formulated as a standard optimal control problem, the model determines the optimal electricity consumption path under three regimes: no storage, non-binding storage and binding storage capacities.

The no storage case occurs because the potential welfare gain from transferring energy from peak production is outweighed by conversion losses. Consumer demand is met by combining intermittent renewable generation with dispatchable energy. At each instant, consumption follows renewable output when it is high and is supplemented by dispatchable energy when renewable production falls short. This regime first illustrates how dispatchable generation, together with demand-side adjustment, can accommodate renewable variability. Second, it provides a baseline for assessing the welfare effects of adding storage, particularly since a higher share of renewable energy substituting dispatchable generation reduces consumer welfare.

In the second regime, storage is abundant. Unlike the previous case, where all consumption is directly supplied by renewable and dispatchable energy, here the planner can shift energy from periods of high renewable production to periods of scarcity without hitting the storage limit. This results in a consumption path with two distinct levels: a lower level when renewable output is insufficient and is supplemented by stored and dispatchable energy, and a higher level when renewable production is abundant and energy is being stored. This regime demonstrates that storage can partially offset the variability of renewable energy and reduce welfare losses as the share of renewables replacing dispatchable generation increases.

Finally, we consider the case when storage is used but reaches its capacity limit. In this case, the social planner would like to shift energy from periods of high renewable output to periods of low output, but the storage constraint prevents a full transfer. As a result, the levels of consumption during low and high demand periods are determined not only by renewable and dispatchable supply but also by the storage limit. The lower consumption level falls below the threshold observed when storage is unconstrained, while the higher level reflects the maximum energy that can be stored. This regime shows how binding storage capacity restricts the planner's ability to smooth consumption over time, with the optimal consumption path and storage strategy jointly shaped by dispatchable energy, renewable capacity, and storage size. Furthermore, it shows that a moderate expansion of storage capacity can fully offset the welfare losses associated with substituting dispatchable generation with renewables.

Taken together, the three regimes clarify that while consumers inevitably bear part of the adjustment burden from renewable variability, this burden can be alleviated through storage. In addition, each regime is associated with a distinct level of consumer welfare, allowing us to directly assess how variations in energy mixes, including storage capacities, shape consumption patterns and overall consumer welfare.

The rest of the paper is organized as follows. In Section 2, we present the main features of the model followed by the definition of the optimal electricity consumption path in Section 3. We then characterize this path under three regimes: no storage (Section 4), non-binding storage (Section 5), and binding storage (Section 6). A welfare analysis across different energy mixes is provided in Section 7 and Section 8 discuss some broader implications of our results. The paper concludes in Section 9. All proofs are relegated to the appendices.

2. The model

We consider a social planner who is concerned with the optimal allocation of electricity for consumption in an economy endowed with three technologies: dispatchable power plants, renewable generators and energy storage systems.

The consumption side is represented by a consumer[2] who derives utility from electricity consumed over a period of time which we normalize to the interval $[0, 1]$. This normalization aligns with the temporal cycle of the renewable energy production, as explained later in this section. Our focus is therefore on analyzing the consumer's *electricity consumption path* over $[0, 1]$. We assume that the utility, U, obtained from an entire consumption path, $x(\cdot)$, is the integral over the time interval $t \in [0, 1]$ of the instantaneous utility, $u(x(t))$, where $x(t)$ is the instantaneous electricity consumption. Given the short duration of the cycle, intertemporal discounting is considered negligible, and the utility function is specified as:

$$U(x(\cdot)) = \int_0^1 u(x(t))\, dt \tag{1}$$

We model $u(x)$ as an increasing and concave function that satisfies the Inada conditions: $u'(x) > 0$, $\lim_{x \to 0} u'(x) = \infty$, $\lim_{x \to \infty} u'(x) = 0$ and $u''(x) < 0$.

[2]This representative agent assumption is formulated for explanatory simplicity. Section 8.3 shows that, within our specific problem, this framework is equivalent to a setting involving several heterogeneous consumers.

As we do not impose any functional form on consumption dynamics, the consumer is, in principle, free to vary her consumption along its path. However, it can be observed that the consumer prefers a constant path. Since u is concave, Jensen's inequality implies that a constant consumption path equal to the mean, x^{cst}, of any fluctuating consumption path $x(\cdot)$ yields at least as much utility as the fluctuating path itself.

$$U\left(x^{cst}\right) \geq U\left(x\left(\cdot\right)\right), \text{ where } x^{cst} = \int_0^1 x\left(t\right) dt \tag{2}$$

Nevertheless, due to fluctuations in renewable production, it may not always be possible for the consumer to achieve a perfectly constant stream of consumption. In such cases, it is useful to consider the intertemporal marginal rate of substitution (MRS) between any two periods t and t'. In our model, the intertemporal MRS does not depend on the entire consumption path but only on the consumption levels at times t and t':

$$MRS_{x(t)/x(t')} = \frac{u'\left(x\left(t'\right)\right)}{u'\left(x\left(t\right)\right)}, \quad \forall t, t' \in [0,1] \tag{3}$$

We hence use the MRS in our analysis to characterize how the consumer is willing to substitute consumption intertemporally in response to fluctuations.

To ensure the provision of electricity required for consumption, the social planner considers three technologies, each with capacities fixed at an exogenous level.

The first is a *renewable technology* with installed capacity R. The volume of electricity it produces varies over time due to uncontrollable natural factors such as wind speed or solar radiation. To model this variability, we define a time-varying unit productivity function, $\epsilon\left(t\right)$, which represents the proportion of a unit of installed capacity utilized for electricity production at each instant. We assume that this renewable productivity exhibits a peak, ϵ_M, around which it varies in a repetitive cycle over time. To simplify the analysis, we normalize the minimum of $\epsilon\left(t\right)$ to zero so that $\epsilon\left(t\right) \in [0, \epsilon_M]$. We also normalize the duration of the cycle to 1. This also explains the earlier alignment of the temporal cycle of the consumer's electricity consumption to $[0,1]$. More formally, if $T_M \in [0,1]$ denotes the time at which renewable production reaches its peak, we have $\epsilon\left(0\right) = \epsilon\left(1\right) = 0$ and $\forall t \in (0, T_M)$, $\epsilon'\left(t\right) > 0$ while for $\forall t \in (T_M, 1)$, $\epsilon'\left(t\right) < 0$. The instantaneous volume of electricity production from this technology at any time t is therefore $\epsilon\left(t\right) R$ so that the total volume of renewable production over the time period $[0,1]$ is given by $R \int_0^1 \epsilon\left(t\right)\, dt$.

The second technology is a *dispatchable technology*, where dispatchability refers to the ability to control the energy resource, allowing electricity production to be adjusted freely at any instant.[3] Given that the interval $[0,1]$ is a normalized representation of a

[3] This is characteristic of conventional power plants such as nuclear, coal, and gas, where fuel input can be controlled to adjust electricity output.

short-term cycle in our analysis and that the installed capacity of the technology is fixed, we assume it generates a total volume D of electricity over the period. So, while this total volume can be allocated flexibly over time, the total quantity dispatched across the interval must be equal to D. Letting $d(t)$ denote the instantaneous dispatch at time t, this constraint is formalized as:

$$\int_0^1 d(t)\ dt = D \tag{4}$$

We also have an *energy storage technology* of an installed capacity S that does not produce electricity but rather stores it for later use. We denote by $s^+(t)$ the volume of energy supplied to the storage system at time t and $s^-(t)$ the volume withdrawn from it. The storage process is subject to conversion inefficiencies: it takes $\sigma^+ > 1$ units of electricity to store one unit of energy and only a fraction $\sigma^- \in (0,1)$ of each unit withdrawn from storage is effectively usable for consumption. Accordingly, the overall round-trip efficiency is given by $\frac{\sigma^-}{\sigma^+} < 1$, capturing the total energy losses incurred in the storage and discharge process. To ensure temporal feasibility over the cycle, we assume that the storage system is empty at both the beginning and end of the period. This implies that the net accumulation of stored energy over the interval is zero:

$$\int_0^1 \big(s^+(t) - s^-(t)\big)\, dt = 0 \tag{5}$$

Moreover, the planner is constrained by the system's capacity, which limits the total volume of electricity that can be allocated to storage over the period:

$$\int_0^1 s^+(t)\, dt \leq S \tag{6}$$

Also, storage decisions are not tied to strict chronological ordering. It may be optimal, for example, to withdraw energy early in the cycle—before any has technically been stored—if this helps achieve a smoother consumption path. This apparent paradox reflects the stationarity of the model where time represents position within a cycle rather than absolute sequence, and all decisions are made consistently within this closed temporal structure.

Finally, we denote by $e = (D, R, S) \in \mathcal{E}$ the energy mix which represents the short-term exogenous capacity composition of the supply side. Since our analysis focuses on the short-term consumption adjustments induced by fluctuations in renewable production, we introduce two additional assumptions on this set of parameters. These assumptions ensure that neither the storage capacity nor the available dispatchable energy is sufficient to fully cover the fluctuations in electricity production.

The first condition simply says that the maximum storage capacity, including conversion losses, is bounded above by the total renewable energy produced during the cycle and available for storage. This leads to the following constraint:

$$\sigma^+ S \leq R \int_0^1 \varepsilon(t)\, dt \tag{7}$$

The second condition rules out cases in which dispatchable energy is sufficiently abundant to offset fluctuations caused by renewable production. Since the consumer is averse to fluctuations in electricity consumption (see Eq.(2)), the planner must, in such cases, be able to deliver a constant flow of consumption c throughout the entire period. Moreover, this constant flow must be greater than the maximum renewable production, i.e., $c > \varepsilon_M R$. However, feasibility over the whole cycle requires that the total quantity of dispatchable energy covers this constant consumption path net of renewable production so that: $D = c - R\int_0^1 \varepsilon(t)\, dt$[4]. Since $c > \varepsilon_M R$, this case is excluded if we assume:

$$D \leq R \int_0^1 (\varepsilon_M - \varepsilon(t))\, dt \tag{8}$$

3. The optimal electricity consumption path

The social planner faces a constrained intertemporal allocation problem: how to distribute limited energy resources—dispatchable production, intermittent renewable output, and imperfectly efficient storage—so as to maximize the representative consumer's utility from electricity consumption over the cycle $[0, 1]$. This optimal consumption path solves:

$$W(e) = \max_{(d(t), s^+(t), s^-(t))} \int_0^1 u(\underbrace{d(t) - \sigma^+ s^+(t) + \sigma^- s^-(t) + \varepsilon(t) R}_{x(t)}) dt \tag{9}$$

subject to the 3 iso-perimetric constraints given by Eqs.(4),(5) and (6). Moreover, each of these constraints can be replaced by a differential equation on $[0, 1]$ with terminal state conditions. Hence, for Eqs.(4),(5) and (6), we respectively obtain:

$$\begin{cases} \int_0^1 d(t)dt = D \\ \int_0^1 (s^+(t) - s^-(t))\, dt = 0 \\ \int_0^1 s^+(t)dt \leq S \end{cases} \Leftrightarrow \begin{cases} \dot{D}(t) = -d(t),\ D(0) = D,\ D(1) = 0 \\ \dot{\Sigma}(t) = s^+(t) - s^-(t),\ \Sigma(0) = \Sigma(1) = 0 \\ \dot{S}(t) = -s^+(t),\ S(0) = S,\ S(1) \geq 0 \end{cases} \tag{10}$$

This leads to a standard optimal control problem with 3 state variables. If we denote by $\lambda_D(t)$, $\lambda_\Sigma(t)$ and $\lambda_S(t)$ the associated co-states, the Hamiltonian writes:

$$\mathcal{H} = u(x(t)) - \lambda_D(t)d(t) + \lambda_\Sigma(t)\left(s^+(t) - s^-(t)\right) - \lambda_S s^+(t) \tag{11}$$

Moreover, if we denote respectively by $\mu_d(t)$, $\mu_{s^+}(t)$ and $\mu_{s^-}(t)$ the multipliers associated to the non-negativity conditions on $d(t), s^+(t)$ and $s^-(t)$, the Lagrangian associated to this Hamiltonian becomes:[5]

[4]Recall that we work on $[0, 1]$, so that $\int_0^1 c\, dt = c$.

[5]Since we have assumed that $\lim_{x\to 0} u'(x) = +\infty$, we do not consider the constraint $x \geq 0$. It can easily be shown that this one is always satisfied.

$$\mathcal{L} = \mathcal{H} + \mu_d(t)d(t) + \mu_{s^+}(t)s^+(t) + \mu_{s^-}(t)s^-(t) \tag{12}$$

Let us now turn to the optimality conditions starting with the dynamics of the co-states. As is usual with isoperimetrical constraints, the co-states are constant and using the transversality conditions, we have:

$$\dot{\lambda}_D(t) = -\frac{\partial \mathcal{L}}{\partial D} = 0,\ \lambda_D(1) \text{ free} \tag{13}$$
$$\dot{\lambda}_\Sigma(t) = -\frac{\partial \mathcal{L}}{\partial \Sigma} = 0,\ \lambda_\Sigma(1) \text{ free} \tag{14}$$
$$\dot{\lambda}_S(t) = -\frac{\partial \mathcal{L}}{\partial S} = 0,\ \lambda_S(1)S(1) = 0,\ \lambda_S(1), S(1) \geq 0 \tag{15}$$

Hereafter, we consider the co-states as constant. It follows that the control variables are optimal if we have:

$$\frac{\partial \mathcal{L}}{\partial d} = u'(x(t)) - \lambda_D + \mu_d(t) = 0 \tag{16}$$
$$\frac{\partial \mathcal{L}}{\partial s^+} = -\sigma^+ u'(x(t)) + \lambda_\Sigma - \lambda_S + \mu_{s^+}(t) = 0 \tag{17}$$
$$\frac{\partial \mathcal{L}}{\partial s^-} = \sigma^- u'(x(t)) - \lambda_\Sigma + \mu_{s^-}(t) = 0 \tag{18}$$

and if we satisfy the associated slackness conditions:

$$\mu_d(t)d(t) = 0,\ \mu_d(t) \geq 0,\ d(t) \geq 0 \tag{19}$$
$$\mu_{s^+}(t)s^+(t) = 0,\ \mu_{s^+}(t) \geq 0,\ s^+(t) \geq 0 \tag{20}$$
$$\mu_{s^-}(t)s^-(t) = 0,\ \mu_{s^-}(t) \geq 0,\ s^-(t) \geq 0 \tag{21}$$

Moreover, with regard to this optimization problem, we can immediately observe that all the constraints are linear. So, if we can show that the Hamiltonian is concave with respect to the control and the stock variables, we obtain the following Mangasarian-type of sufficient conditions:

Proposition 1. *If, for a continuous and piecewise differentiable control path given by* $(d(t), s^+(t), s^-(t))$*, there exist constant co-states* $(\lambda_D, \lambda_\Sigma, \lambda_S)$*, and continuous and piecewise differentiable multipliers* $(\mu_d(t), \mu_{s^+}(t), \mu_{s^-}(t))$ *that satisfy Eqs.(10), (15) and (16) to (21), then* $(d(t), s^+(t), s^-(t))$ *is a solution to the previous problem.*

From the solution to the optimization problem, several features of the optimal allocation follow immediately. In particular, consumption may be constant over some time intervals and certain combinations of controls are ruled out.

First, suppose that dispatchable energy is used over some interval (t, t'), i.e., $d(t) > 0$. Then, by the slackness condition (19), we have $\mu_d(t) = 0$ which implies that consumption $x(t)$ is constant over this interval since $x(t) = u^{-1}(\lambda_D)$ from Eq.(16). Similarly, if storage is used over another interval (t, t'), it follows from the corresponding slackness condition, (20) or (21), that either $\mu_{s^+}(t) = 0$ or $\mu_{s^-}(t) = 0$. In that case, consumption is again constant as the co-states in Eqs.(17) and (18) are constant.

Also, we obtain further restrictions: if storage and discharge occur simultaneously, one can always improve the allocation by setting the smallest of the two to zero and adjusting the other so as to preserve net storage. This reduces conversion losses and therefore increases consumption over the cycle. Finally, if production comes from dispatchable generation, storing part of it ($s^+ > 0$) would convert dispatchable energy $d(t)$ into stored electricity recovered with efficiency $\sigma^-/\sigma^+ < 1$, resulting in a net loss. This strategy is clearly dominated.

The next Lemma summarizes these observations.

Lemma 1. *On any interval (t, t'), the following properties hold:*
(i) If dispatchable production is used, then $\mu_d(t) = 0$ and consumption is constant with $x(t) = \bar{c}$.
(ii) If storage is used, then either $\mu_{s^+}(t) = 0$ or $\mu_{s^-}(t) = 0$ and consumption is again constant.
(iii) Storage and discharge cannot occur simultaneously: $s^+(t)\, s^-(t) = 0$.
(iv) Dispatchable production and storage cannot occur simultaneously: $s^+(t)\, d(t) = 0$.

4. Conversion losses vs. storage

This section focuses on scenarios without energy storage due to conversion losses and low fluctuations in renewable energy production. The main objective is to characterize the conditions under which this case arises and, conversely, those under which storage becomes part of the optimal electricity allocation problem.

We have seen in the model section that some energy mixes e are ruled out. In particular, we exclude the case in which storage is not needed because dispatchable energy is sufficiently large relative to renewable generation to sustain a constant consumption stream c^a that lies above the peak of renewable production, $\varepsilon_M R$, thereby fully compensating renewable-induced fluctuations in consumption. But this raises the following question: if a feasible constant consumption path does not reach the peak of renewable generation, i.e., $c^a < \varepsilon_M R$, does storage necessarily take place? This is where the conversion losses associated with storage come into play. In fact, one unit of electricity stored—for example, during a production peak—yields only a potential electricity consumption of $\frac{\sigma^-}{\sigma^+} < 1$ units when it is withdrawn. This prompts the question of whether it is in the interest of consumers to transfer electricity from the peak of renewable generation (without dispatchable production), $\varepsilon_M R$, to a lower constant consumption path, $c^a < \varepsilon_M R$, supported by the use of dispatchable energy. Recall that the marginal rate of substitution, $MRS_{c^a/\epsilon_M R} = \frac{u'(\varepsilon_M R)}{u'(c^a)}$, measures the increase in consumption at c^a that keeps utility constant following a one-unit decrease in consumption at the peak production, $\varepsilon_M R$. Hence, if the quantity of electricity that can actually be transferred due to conversion losses, $\frac{\sigma^-}{\sigma^+}$, is less than $MRS_{c^a/\epsilon_M R}$, no storage occurs. We now introduce the level of electricity consumption $\bar{x} < \varepsilon_M R$ given by:

$$MRS_{\bar{x}/\epsilon_M R} = \frac{u'(\varepsilon_M R)}{u'(\bar{x})} = \frac{\sigma^-}{\sigma^+} \Leftrightarrow \bar{x} = (u')^{-1}\left(\frac{\sigma^+}{\sigma^-} u'\left(\varepsilon_M R\right)\right) \tag{22}$$

Since $MRS_{c/\epsilon_M R}$ increases with c, $\bar{x}$ can be interpreted as the highest consumption plateau at which storage occurs. Conversely, if the optimal consumption path $x(t)$ solving program (16) satisfies $x(t) \geq \bar{x}$, no storage takes place.

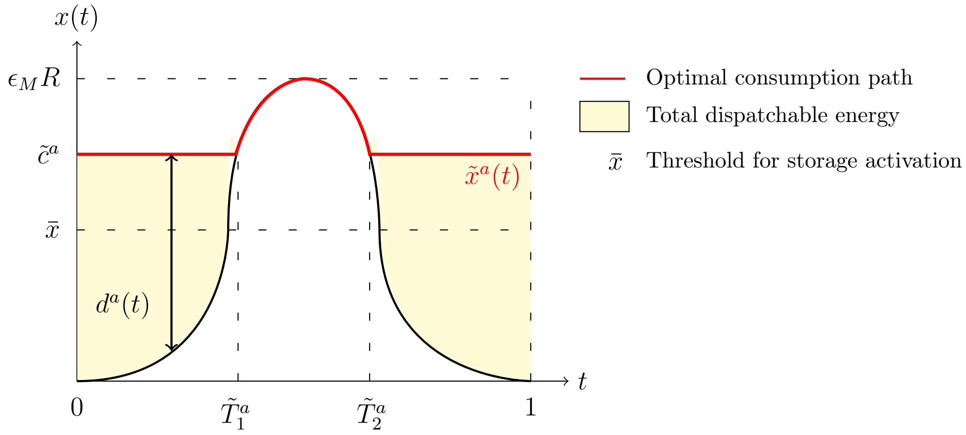

Figure 1: Consumption without storage

Two open questions remain: (1) what is the optimal consumption path in this case and (2) under which conditions does it satisfy $x^a(t) \geq \bar{x}$? As long as the dispatchable technology is used, Lemma 1(i) ensures that the consumption path is constant at c^a. We can further conjecture that without the dispatchable generation, the renewable peak is absorbed (see Fig. 1). The corresponding consumption path is given by:

$$\forall t \in [0,1], \;\; x(c^a, t) = \max\{c^a, \varepsilon(t)R\} \tag{23}$$

In other words, the use of dispatchable energy supplements renewable production to maintain this constant consumption path as long as the required amount exceeds the energy produced by the renewable technology, i.e.,

$$\forall t \in [0,1], \;\; d(c^a, t) = \max\{c^a - \varepsilon(t)R, 0\} \tag{24}$$

This consumption plateau c^a in nevertheless an endogenous variable since the sum of dispatchable energy used at each $t \in [0,1]$ must be equal to the total amount D of dispatchable energy allocated to the cycle (see the yellow area in Fig. 1). In other words, c^a satisfies:

$$\int_0^1 d(c^a, t)\, dt = \int_0^1 \max\{c^a - \varepsilon(t)R, 0\}\, dt = D \tag{25}$$

Now, recall that no storage occurs if $\forall t \in [0,1]$, $x(t) \geq \bar{x}$. This simply means, by Eq.(23), that the solution $\tilde{c}^a$ to Eq.(25) is larger than $\bar{x}$. As $\int_0^1 \max\{c^a - \varepsilon(t)R, 0\}\, dt$ increases with c^a, a necessary (and sufficient) condition ensuring that no storage occurs is given by:

$$\int_0^1 \max\{\bar{x} - \varepsilon(t)R, 0\}\, dt \leq D \tag{26}$$

To sum up this discussion, we can state the following:

Proposition 2. *If* $D \geq \int_0^1 \max\{\bar{x} - \varepsilon(t)R, 0\}\, dt$,
(i) there exists a unique solution $\tilde{c}^a$ *to Eq.(25) with the property that* $\tilde{c}^a \geq \bar{x}$.
(ii) the optimal electricity consumption path that solves program (57) is given by $\forall t \in [0,1]$, $\tilde{x}^a(t) = \max\{\tilde{c}^a, \varepsilon(t)R\}$.
(iii) the path $\tilde{x}(t)$ *requires no storage:* $\forall t \in [0,1]$, $(\tilde{s}^-)^a(t) = (\tilde{s}^+)^a(t) = 0$, *but only the additional use of dispatchable energy, given by* $\forall t \in [0,1]$, $\tilde{d}^a(t) = \max\{\tilde{c}^a - \varepsilon(t)R, 0\}$.

To prove this proposition, as shown in Appendix B, we first demonstrate that there exists a unique consumption plateau $\tilde{c}^a \geq \bar{x}$ that solves Eq.(25). We then verify that the induced paths, $\tilde{x}^a(t)$ and $\tilde{d}^a(t)$, satisfy the sufficient optimality conditions provided by Proposition 1.

Also, according to (i) of Proposition 2, the consumption plateau that solves Eq.(25) is a function of the amounts of dispatchable energy D and renewable energy R available during the period $[0,1]$. Therefore, it is interesting to know the effect of these parameters on this plateau $\tilde{c}^a(D,R)$. While the direction of these effects is intuitive—an increase in either D or R raises the consumption plateau, $\tilde{c}^a$—we explicitly derive these comparative statics here. This derivation will be useful later in Section 7 where we study how substituting dispatchable energy with renewable energy affects consumer welfare.

To carry out this derivation, we define the times at which consumption switches across the plateau, as illustrated in Fig. 1. Using the properties of $\varepsilon(t)$, we define $\tilde{T}_i^a(D,R) = \varepsilon^{-1}\left(\frac{\tilde{c}^a(D,R)}{R}\right)$ as the two solutions in t to $\varepsilon(t)R = \tilde{c}^a(D,R)$ for $i = 1,2$ and with $\varepsilon'(\tilde{T}_1^a(D,R)) > 0$ and $\varepsilon'(\tilde{T}_2^a(D,R)) < 0$. With these additional notations, Eq.(25) can be rewritten as:

$$\int_0^{\tilde{T}_1^a(D,R)} (\tilde{c}^a(D,R) - \varepsilon(t)R)\, dt + \int_{\tilde{T}_2^a(D,R)}^1 (\tilde{c}^a(D,R) - \varepsilon(t)R)\, dt - D = 0 \qquad (27)$$

and becomes a differentiable identity in (D,R). A simple calculation (see Appendix F) yields the results presented in Table 1. As expected, both derivatives are positive. However, their magnitudes differ. An increase in dispatchable energy is spread over the intervals $[0, T_1^a]$ and $[T_2^a, 1]$, which explains their presence in the derivatives. An increase in renewable energy increases total energy production and redistributes dispatchable energy over the segments where it is used.

Table 1: Comparative statics without storage

	∂D	∂R
$\partial \tilde{c}^a$	$\frac{1}{1+\tilde{T}_1^a(D,R)-\tilde{T}_2^a(D,R)} > 0$	$\frac{\int_0^{\tilde{T}_1^a(D,R)} \varepsilon(t)dt+\int_{\tilde{T}_2^a(D,R)}^1 \varepsilon(t)dt}{1+\tilde{T}_1^a(D,R)-\tilde{T}_2^a(D,R)} > 0$

5. Optimal consumption with abundant storage

Our previous discussion characterizes the case in which no storage is used, as given by condition (26). If the opposite holds, i.e.,

$$\int_0^1 \max\left\{\bar{x} - \varepsilon(t)R, 0\right\} dt > D \text{ with } \bar{x} \text{ given by } \bar{x} = (u')^{-1}\left(\frac{\sigma^+}{\sigma^-}u'\left(\varepsilon_M R\right)\right) \tag{28}$$

we can expect that storage is used. This raises the question of what the optimal electricity consumption path looks like in this case, particularly when storage capacity is abundant.

From Lemma 1, we know that the use of dispatchable and/or stored energy cannot occur simultaneously with energy storage. Moreover, after accounting for conversion losses, the remaining energy can be treated as an additional stock of dispatchable energy. This implies that stored and dispatchable energy are used together when renewable energy production is low, whereas energy storage occurs during periods of peak renewable production. Lemma 1 also indicates that consumption is time-invariant whenever dispatchable energy is used or energy is stored. Consequently, there are now two endogenous consumption plateaus, $\tilde{c}_1^b$ and $\tilde{c}_2^b$. The first corresponds to periods of low renewable energy production, relying mainly on dispatchable and stored energy, while the second corresponds to periods of high renewable energy production, achieved through storage (see Fig.2)

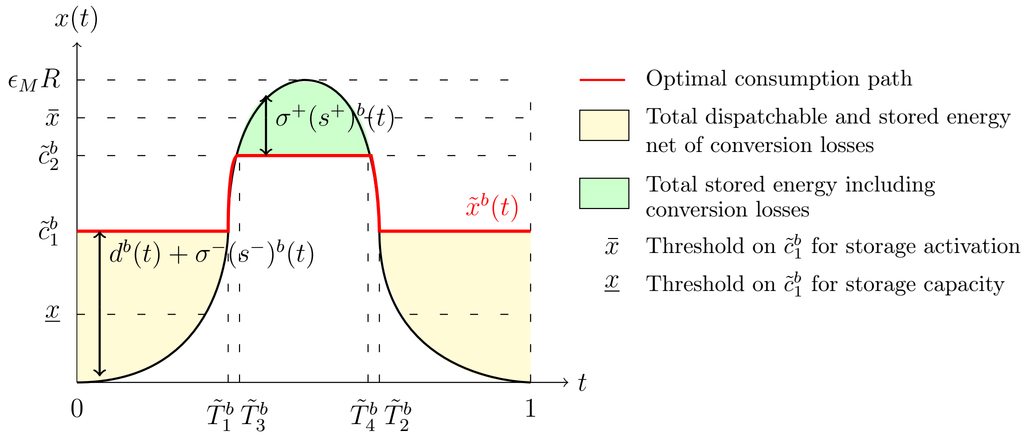

Figure 2: Consumption with abundant storage

More formally, this suggests that the optimal consumption path, illustrated in Fig.2, takes the form:

$$\forall t \in [0,1]\,,\ \ x\left(c_1^b, c_2^b, t\right) = \min\left\{\max\{c_1^b, \varepsilon(t)R\}, c_2^b\right\} \text{ with } c_1^b < c_2^b \tag{29}$$

with the following features:

- supplementing renewable generation when it is low, i.e., $\varepsilon(t)R < c_1^b$, using dispatchable and stored energy, given by:

$$\forall t \in [0,1]\,,\ d(c_1^b, t) + \sigma^- s^-(c_1^b, t) = \max\left\{c_1^b - \varepsilon(t)R, 0\right\} \tag{30}$$

- storing renewable energy when its production is high, i.e., $\varepsilon(t)R > c_2^b$, with the storage path given by:

$$\forall t \in [0,1], \;\; s^+(c_2^b, t) = \tfrac{1}{\sigma^+} \max\left\{\varepsilon(t)R - c_2^b, 0\right\} \tag{31}$$

- continuous adjustment between the two levels during which only renewable energy is consumed.

The question that remains open is the identification of the two endogenous consumption plateaus, c_1^b and c_2^b.

We first argue that there exists a relationship between c_1^b and c_2^b when storage capacity is not binding. Since energy is transferred from the consumption plateau, c_2^b, to the lower plateau, c_1^b, without any capacity constraint, the social planner ensures that the marginal rate of substitution between the two consumption levels is equal to the ratio of conversion losses, as follows:

$$MRS_{c_1^b/c_2^b} = \frac{u'\left(c_2^b\right)}{u'\left(c_1^b\right)} = \tfrac{\sigma^-}{\sigma^+} \tag{32}$$

Otherwise, the planner has an incentive to adjust these two consumption levels to improve the consumer's situation, either by increasing or decreasing storage. Therefore, the following relationship between c_1^b and c_2^b must hold:

$$c_2^b = f(c_1^b) = (u')^{-1}\left(\tfrac{\sigma^-}{\sigma^+} u'(c_1^b)\right) \tag{33}$$

We can even note that this function is increasing since:

$$f'(c_1^b) = \tfrac{\sigma^-}{\sigma^+}\left(u''\left((u')^{-1}\left(\tfrac{\sigma^-}{\sigma^+} u'(c_1^b)\right)\right)\right)^{-1} u''(c_1^b) > 0 \tag{34}$$

The existence of this relationship has several consequences. As expected, we can claim that $c_2^b > c_1^b$ since $MRS_{c_1^b/c_2^b}$ is decreasing in c_2^b and $MRS_{c_1^b/c_1^b} = 1 > \frac{\sigma^-}{\sigma^+}$. This confirms the idea that when storage occurs, the consumer increases her consumption to partially limit conversion losses. Moreover, consistency requires that the consumption level at which storage occurs, c_2^b, is lower than the maximum renewable energy production, i.e., $c_2^b < \varepsilon_M R$. Since $f(c_1^b)$ is increasing, Eq.(22) implies that $c_1^b < \bar{x}$. This means that the consumption plateau at which dispatchable energy is used is lower than smallest consumption plateau, $\bar{x}$, introduced in the previous section for which no storage occurs. This validates the idea that the condition given by Eq.(28) is a minimal one ensuring the existence of storage.

It now remains to show how the endogenous plateau c_1^b is determined. At this level of consumption, renewable energy production is insufficient and the resulting gap is instantaneously covered by a combination of dispatchable and stored energy (net of the conversion losses), in the amount $\max\left\{c_1^b - \varepsilon(t)R, 0\right\}$, $\forall t \in [0,1]$ (see Eq.(30)). These energy sources, however, are available only in limited quantities over the renewable energy cycle. Dispatchable energy over $[0,1]$ is limited to D while under our stationarity assumption, stored energy is limited by the provision made during peak renewable production. Using Eqs.(30) and (31), this leads to the following condition:

$$\int_0^1 \max\left\{c_1^b - \varepsilon(t)R, 0\right\} dt = D + \sigma^- \int_0^1 \tfrac{1}{\sigma^+} \max\left\{\varepsilon(t)R - f(c_1^b), 0\right\} dt \qquad (35)$$

However, recall that the main argument of this section is based on the assumption that the storage capacity is not binding. Using Eqs.(31) and (33), this requires that $\int_0^1 s^+(f(c_1^b), t)dt < S$. So let us now introduce a second threshold $\underline{x}$ for which the storage capacity is saturated, i.e.,

$$\int_0^1 s^+(f(\underline{x}), t)dt = \int_0^1 \max\left\{\varepsilon(t)R - f(\underline{x}), 0\right\} = S \qquad (36)$$

It can be shown that this unique threshold satisfies $\underline{x} < \bar{x}$ and that for $c_1^b \geq \underline{x}$, the storage capacity is not strictly binding. This observation yields a condition on the parameters of the model that ensures the capacity constraint is never binding. This condition states that if the sum of dispatchable energy, D, and storage capacity net of conversion costs, $\sigma^- S$, is greater than the amount of energy required to sustain a consumption plateau at $\underline{x}$, then the storage capacity is not binding. This condition can be written as:

$$\int_0^1 \max\left\{\underline{x} - \varepsilon(t)R, 0\right\} dt \leq D + \sigma^- S \qquad (37)$$

The following proposition summarizes this discussion.

Proposition 3. *If the conditions given by Eqs.(28) and (37) are verified, then*
(i) there exists a unique solution, $\tilde{c}_1^b$, to Eq.(35) with the property that $\underline{x} \leq \tilde{c}_1^b < \bar{x}$. The second consumption plateau is given by $\tilde{c}_2^b = (u')^{-1}\left(\frac{\sigma_-}{\sigma_+} u'(\tilde{c}_1^b)\right)$.
(ii) the optimal electricity consumption path which solves program (57) is given by $\forall t \in [0,1]$, $\tilde{x}^b(t) = \min\left\{\max\left\{\tilde{c}_1^b, \varepsilon(t)R\right\}, \tilde{c}_2^b\right\}$.
(iii) the optimal storage strategy occurs during peak renewable production. It is given by $\forall t \in [0,1]$, $(\tilde{s}^+)^b(t) = \frac{1}{\sigma^+}\max\left\{\varepsilon(t)R - \tilde{c}_2^b, 0\right\}$ and the storage capacity is non-binding.
(iv) there are several ways to allocate dispatchable and stored energy as long as $\forall t \in [0,1]$, $\tilde{d}^b(t) + \sigma^-(\tilde{s}^-)^b(t) = \max\left\{\tilde{c}_1^b - \varepsilon(t)R, 0\right\}$, $\int_0^1 \tilde{d}^b(t)dt = D$ and $\int_0^1 (\tilde{s}^-)^b(t)dt = \int_0^1 \tilde{s}^+(t)dt$.

Point (iv) indicates that the solution to the optimization problem is not unique. This is not surprising: once stored, electricity effectively functions as an alternative source of dispatchable energy. The two types of energy can therefore be used indifferently to reach the consumption plateau $\tilde{c}_1^b$ corresponding to low renewable energy production. In Appendix D, we show that any continuous and positive selection satisfying the conditions given by (iv) of Proposition 3 can be part of the optimal solution.

As in the previous section, we now characterize the effect of a change in dispatchable or renewable energy on the two consumption plateaus, $c_1^b(D,R)$ and $c_2^b(D,R)$. To do this, we introduce the four switching times given respectively by $\tilde{T}_i^b(D,R) = \varepsilon^{-1}\left(\frac{\tilde{c}_1^b(D,R)}{R}\right)$, $i=1,2$ and $\tilde{T}_i^b(D,R) = \varepsilon^{-1}\left(\frac{\tilde{c}_2^b(D,R)}{R}\right)$, $i=3,4$ (see Fig.2). Using these notations, Eq.(35) now writes:

$$\int_0^{\tilde{T}_1^b(D,R)} \left(\tilde{c}_1^b(D,R) - \varepsilon(t)R\right) dt + \int_{\tilde{T}_4^b(D,R)}^1 \left(\tilde{c}_1^b(D,R) - \varepsilon(t)R\right) dt$$
$$+\tfrac{\sigma^-}{\sigma^+}\int_{\tilde{T}_2^b(D,R)}^{\tilde{T}_3^b(D,R)} \left(\tilde{c}_2^b(D,R) - \varepsilon(t)R\right) dt - D = 0 \qquad (38)$$

and by Eq.(33), we know that:

$$\tilde{c}_2^b(D,R) = f(\tilde{c}_1^b(D,R)) = (u')^{-1}\left(\tfrac{\sigma^-}{\sigma^+}u'(\tilde{c}_1^b(D,R))\right) \qquad (39)$$

The results described in Table 2 follow from the differentiation of these two equations with respect to D and R (see Appendix F). Unsurprisingly, we again find that $\tilde{c}_1^b(D,R)$ increases in both D and R simply because, in each case, more electricity is available. The results for $\tilde{c}_2^b(D,R)$ follow from the positive relationship between $\tilde{c}_2^b(D,R)$ and $\tilde{c}_1^b(D,R)$, i.e. $f'(\tilde{c}_1^b) > 0$ (see Eq.(34)). We use these results in the discussion of the consumer's welfare in Section 7.

Table 2: Comparative statics with non binding storage

	∂D	∂R
$\partial \tilde{c}_1^b$	$\frac{u''(\tilde{c}_2^b)}{(1+\tilde{T}_1^b-\tilde{T}_2^b)u''(\tilde{c}_2^b)+\left(\frac{\sigma^-}{\sigma^+}\right)^2(\tilde{T}_4^b-\tilde{T}_3^b)u''(\tilde{c}_1^b)} > 0$	$\frac{\left(\int_0^{\tilde{T}_1^b}\varepsilon(t)dt+\int_{\tilde{T}_2^b}^1\varepsilon(t)d+\frac{\sigma^-}{\sigma^+}\int_{\tilde{T}_3^b}^{\tilde{T}_4^b}\varepsilon(t)dt\right)u''(\tilde{c}_2^b)}{(1+\tilde{T}_1^b-\tilde{T}_2^b)u''(\tilde{c}_2^b)+\left(\frac{\sigma^-}{\sigma^+}\right)^2(\tilde{T}_4^b-\tilde{T}_3^b)u''(\tilde{c}_1^b)} > 0$
$\partial \tilde{c}_2^b$	$\frac{\frac{\sigma^-}{\sigma^+}u''(\tilde{c}_1^b)}{(1+\tilde{T}_1^b-\tilde{T}_2^b)u''(\tilde{c}_2^b)+\left(\frac{\sigma^-}{\sigma^+}\right)^2(\tilde{T}_4^b-\tilde{T}_3^b)u''(\tilde{c}_1^b)} > 0$	$\frac{\left(\int_0^{\tilde{T}_1^b}\varepsilon(t)dt+\int_{\tilde{T}_2^b}^1\varepsilon(t)d+\frac{\sigma^-}{\sigma^+}\int_{\tilde{T}_3^b}^{\tilde{T}_4^b}\varepsilon(t)dt\right)\frac{\sigma^-}{\sigma^+}u''(\tilde{c}_1^b)}{(1+\tilde{T}_1^b-\tilde{T}_2^b)u''(\tilde{c}_2^b)+\left(\frac{\sigma^-}{\sigma^+}\right)^2(\tilde{T}_4^b-\tilde{T}_3^b)u''(\tilde{c}_1^b)} > 0$

6. Binding storage capacity

This last case is, in some respects, similar to the previous one, since storage occurs in both cases, but here the storage capacity is binding. This suggests that the discussion at the beginning of the previous section, based on Lemma 1 and concerning the properties of the solution, remains applicable. In particular, the general form of the solutions is the same as described by Eqs.(29),(30) and (31). The main difference lies in the construction and the levels of the two consumption plateaus, c_1^c and c_2^c, which differ from those in the previous case. For instance, in Fig. 3, the green area corresponds to the storage capacity augmented by the conversion losses, the lower consumption plateau now lies below $\underline{x}$ and

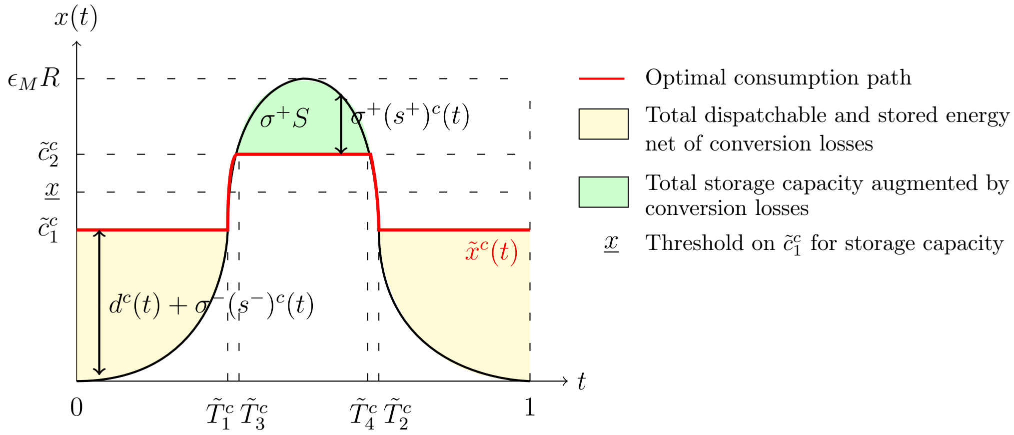

Figure 3: Consumption with binding storage

the relationship between c_1^c and c_2^c is more complex than in the previous section. We therefore start by showing how c_1^c and c_2^c are constructed, which underlies the differences highlighted.

First, note that a binding storage capacity restricts the transfer of electricity consumption over time, notably between the two consumption plateaus c_2^c and c_1^c. This implies that the consumer is still willing to transfer electricity from the high consumption plateau c_2^c to the low consumption plateau c_1^c, but is unable to do so due to the capacity constraint. In other words, we end up with the marginal rate of substitution between c_1^c and c_2^c being below the ratio of the conversion losses. Based on equations (32) and (33), we have:

$$MRS_{c_1^c/c_2^c} = \frac{u'\left(c_2^c\right)}{u'\left(c_1^c\right)} < \tfrac{\sigma^-}{\sigma^+} \text{ or } c_2^c > (u')^{-1}\left(\tfrac{\sigma^-}{\sigma^+}u'(c_1^c)\right) > c_1^c \tag{40}$$

It also implies that the total energy stored at the high consumption plateau c_2^c must equal the storage capacity (see Fig. 3), i.e.:

$$\int_0^1 s^+(c_2^c, t)\, dt = \tfrac{1}{\sigma^+}\int_0^1 \max\left\{\varepsilon(t)R - c_2^c, 0\right\}\, dt = S \tag{41}$$

This equation corresponds to Eq.(36), which, in the previous section, introduced the lowest consumption plateau, c_1^b, for which storage is not binding. First, this implies that c_2^c can be deduced from our previous results by simply setting $c_2^c = f(\underline{x})$, with $f(x)$ given by Eq.(33). Second, since $\underline{x}$ is the threshold for c_1^c below which the storage constraint becomes binding, we must ensure that the lowest consumption plateau satisfies $c_1^c < \underline{x}$.

Finally, to characterize the optimal consumption trajectory, it remains to define c_1^c. This is done by stating that the total dispatchable and stored energy required to sustain the low-consumption plateau (the yellow area in Fig. 3) equals the sum of the dispatchable energy and the storage capacity since the latter is now binding, i.e.:

$$\int_0^1 \tilde{d}^c(t) + \sigma^-(\tilde{s}^-)^c(t)dt = \int_0^1 \max\{\tilde{c}_1^c - \varepsilon(t)R, 0\} dt = D + \sigma^- S \tag{42}$$

Moreover, to ensure that $\tilde{c}_1^c < \underline{x}$, we require the contrary of condition (37), that is:

$$\int_0^1 \max\{\underline{x} - \varepsilon(t)R, 0\} dt > D + \sigma^- S \tag{43}$$

Let us, however, recall that $\underline{x} = f^{-1}(c_2^c)$ and is independent of D (see Eq.(41)). This implies that $\int_0^1 \max\{\underline{x} - \varepsilon(t)R, 0\} dt - \sigma^- S$ may be negative, particularly if S is large, which would contradict $D \geq 0$. In other words, if the storage capacity is too large, it cannot be binding and this last case becomes vacuous. To consistently treat the three cases, we therefore introduce, for the remainder of the paper, an additional restriction on the set $\mathcal{E}$ of energy mixes, given by:

$$\int_0^1 \max\left\{f^{-1}(c_2^c(R,S)) - \varepsilon(t)R, 0\right\} dt > \sigma^- S \text{ with } c_2^c(R,S) \text{ solution to Eq.(41)} \tag{44}$$

The following proposition summarizes our results.

Proposition 4. *If the condition given by Eqs.(43) and (44) are verified, then:*
(i) there exists a unique solution, $\tilde{c}_1^c$, to Eq.(42) with the property that $\tilde{c}_1^c < \underline{x}$. The second consumption plateau is given by $\tilde{c}_2^c = f(\underline{x})$ with $f(x)$ given by Eq.(33).
(ii) the optimal electricity consumption path which solves program (57) is given by $\forall t \in [0,1]$, $\tilde{x}^c(t) = \min\{\max\{\tilde{c}_1^c, \varepsilon(t)R\}, \tilde{c}_2^c\}$.
(iii) the optimal storage strategy is limited by the storage capacity and is given by $\forall t \in [0,1]$, $(\tilde{s}^+)^c(t) = \frac{1}{\sigma^+}\max\{\varepsilon(t)R - \tilde{c}_2^c, 0\}$.
(iv) there are again several ways to affect dispatchable and stored energy as long as $\forall t \in [0,1]$, $\tilde{d}^c(t) + \sigma^-(\tilde{s}^-)^c(t) = \max\{\tilde{c}_1^c - \varepsilon(t)R, 0\}$, $\int_0^1 \tilde{d}^c(t)dt = D$ and $\int_0^1 (\tilde{s}^-)^c(t)dt = S$.

Since storage is limited in this case, we examine how dispatchable energy, renewable generation capacity and storage capacity affect the consumption plateaus $\tilde{c}_1^c(D,R,S)$ and $\tilde{c}_2^c(R,S)$. This again requires the construction of the four switching times, given by $\tilde{T}_i^c(D,R,S) = \varepsilon^{-1}\left(\frac{\tilde{c}_1^c(D,R,S)}{R}\right)$, $i = 1,2$ and $\tilde{T}_i^c(R,S) = \varepsilon^{-1}\left(\frac{\tilde{c}_2^c(R,S)}{R}\right)$, $i = 3,4$ (see Fig.3) and the introduction of the two identities induced by the definitions of $\tilde{c}_1^c$ and $\tilde{c}_2$, respectively. These identities are given by:

$$\int_0^{\tilde{T}_1^c(D,R,S)} \left(\tilde{c}_1^c(D,R,S) - \varepsilon(t)R\right) dt + \int_{\tilde{T}_2^c(D,R,S)}^1 \left(\tilde{c}_1^c(D,R,S) - \varepsilon(t)R\right) dt - D - \sigma^- S = 0 \tag{45}$$

$$\int_{\tilde{T}_3^c(D,R,S)}^{\tilde{T}_4^c(D,R,S)} \max\{\varepsilon(t)R - \tilde{c}_2^c(R,S), 0\} dt - \sigma^+ S = 0 \tag{46}$$

By differentiating these two identities with respect to D, R and S (see Appendix F), we obtain the results presented in Table 3. As expected, $\tilde{c}_1^c$ still increases with both dispatchable energy D and renewable capacity R. Moreover, this plateau rises with storage capacity S. The intuition is straightforward: increasing storage capacity allows more electricity to be stored and since stored energy is fully dispatchable, consumption increases accordingly, i.e., to reach $\tilde{c}_1^c$. When storage is limited, the marginal rate of substitution between $\tilde{c}_1^c$ and $\tilde{c}_2^c$ is no longer determined by the ratio of the conversion losses but by the storage constraint. Thus, $\tilde{c}_2^c$ becomes independent of the level of dispatchable energy and decreases with storage capacity to expand stored electricity. Finally, if renewable capacity increases, so does peak production. With limited storage capacity, the only option is to increase $\tilde{c}_2^c$.

Table 3: Comparative statics with binding storage

	∂D	∂R	∂S
$\partial \tilde{c}_1^c$	$\frac{1}{\tilde{T}_1^c+1-\tilde{T}_2^c} > 0$	$\frac{\int_0^{\tilde{T}_1^c} \varepsilon(t)dt+\int_{\tilde{T}_2^c}^1 \varepsilon(t)dt}{\tilde{T}_1^c+1-\tilde{T}_2^c} > 0$	$\frac{\sigma^-}{\tilde{T}_1^c+1-\tilde{T}_2^c} > 0$
$\partial \tilde{c}_2^c$	0	$\frac{\int_{\tilde{T}_3^c}^{\tilde{T}_4^c} \varepsilon(t)dt}{\tilde{T}_4^c-\tilde{T}_3^c} > 0$	$-\frac{\sigma^+}{\tilde{T}_4^c-\tilde{T}_3^c} < 0$

7. Energy mix and consumers' welfare

This section proceeds in two steps. First, we summarize our previous results in order to construct the consumer welfare function for various energy mixes $e = (D, R, S) \in \mathcal{E}$ and to show that this function is continuously differentiable. This enables us, in a second step, to assess the effect on consumer welfare of substituting dispatchable energy with renewable energy.

7.1. The consumer welfare function

To construct this function, let us briefly review our previous results. We consider an energy mix, $e = (D, R, S) \in \mathcal{E}$, in which fluctuations in renewable generation affect consumption. This is ensured by limiting both the storage capacity (see Eq.(7)) and the availability of dispatchable energy (see Eq.(8)). We even add an additional restriction (see Eq.(44)) ensuring that the storage constraint can be binding.[6] Sections 4 to 6 identify the subsets $\mathcal{E}^a$, $\mathcal{E}^b$ and $\mathcal{E}^c$ which respectively correspond to situations without storage, with sufficiently large storage and with binding storage capacity. If these three subsets form a partition of the set $\mathcal{E}$ of the energy mix under consideration (see preliminary remarks

[6]If this restriction is not satisfied, it simply means that the binding-storage case is empty and that all results associated with this case can be ignored. However, we believe it is appropriate to allow for this situation, as storage capacities are still at an early development stage.

in Appendix G), we can then construct the welfare function $W(e)$ resulting from the program outlined in Section 3.

Moreover, to unify the notation, we introduce a virtual plateau $\tilde{c}_2^a(e)$ in the no-storage case. It is defined by the highest renewable production, i.e., $\forall e \in \mathcal{E}^a$, $\tilde{c}_2(e) = \varepsilon_M R$. This harmless trick enables us to define, for all $e \in \mathcal{E}$, piecewise functions $\tilde{c}_1(e)$ and $\tilde{c}_2(e)$ that describe the two consumption plateaus arising in each of the three subcases. With this notation, the optimal consumption path, for each $e \in \mathcal{E}$, becomes:

$$\forall t \in [0,1], \; \tilde{x}(t;e) = \min\left\{\max\left\{\tilde{c}_1(e), \varepsilon(t)R\right\}, \tilde{c}_2(e)\right\} \tag{47}$$

We can even calculate the consumer's welfare for each energy mix over a renewable cycle. This function is given by:

$$W(e) = \int_0^1 u(\tilde{x}(t;e))\,dt \tag{48}$$

More interestingly, if we can show that $\tilde{c}_1(e)$ and $\tilde{c}_2(e)$ are continuous over the entire domain $\mathcal{E}$, then the same holds for $\tilde{x}(t;e)$ and $W(e)$. We go even one step further. Using the results in Tables 1 to 3, we compute $\partial W(e)$ piecewise and show that this gradient remains globally continuous. More precisely, we have the following:

Lemma 2. *For all $e \in \mathcal{E}$, we can say that:*
(i) the two plateaus, $\tilde{c}_1(e)$ and $\tilde{c}_2(e)$, the consumption process $\tilde{x}(t;e)$ and the consumer's welfare $W(e)$ are continuous functions.
(ii) the welfare function $W(e)$ is at least of class C^1 and its gradient is given by:

$$\partial W(e) = \begin{pmatrix} u'(\tilde{c}_1(e)) \\ \int_0^1 u'\left(\min\left\{\max\left\{\tilde{c}_1(e), \varepsilon(t)R\right\}, \tilde{c}_2(e)\right\}\right)\varepsilon(t)dt \\ \sigma^+ u'(\tilde{c}_1(e)) \max\left\{\frac{\sigma^-}{\sigma^+} - \frac{u'(\tilde{c}_2(e))}{u'(\tilde{c}_1(e))}, 0\right\} \end{pmatrix} \tag{49}$$

This technical lemma leads to an interesting observation: although the different energy sources are substitutes from the consumer's perspective (see Eq.(57)), changes in their availability over the renewable production cycle generate different welfare effects. Indeed, a marginal change in one resource affects the optimal consumption path defined over $[0,1)$ in a different way. An increase in dispatchable capacity provides additional consumption outside peak production times, so its marginal contribution to welfare is evaluated through the marginal utility of the lowest consumption plateau. An increase in renewable capacity increases production at each t and therefore, the consumption path. The effect on welfare is then the "sum" of the different marginal utilities along the optimal consumption path. Lastly, storage capacity matters only when the storage constraint is binding because in this case, $\frac{\sigma^-}{\sigma^+} > \frac{u'(\tilde{c}_2(e))}{u'(\tilde{c}_1(e))}$.

7.2. The welfare effect of energy transition on consumers

Lemma 2 also helps us understand the impact of the energy transition on consumer welfare, particularly in the presence of a storage system. Although we cannot examine the transition process itself within our short-term approach, we can nevertheless study the welfare effects of substituting dispatchable energy with renewable production capacities while keeping total energy supply constant over the renewable production cycle. In this setting, we essentially try to address three questions. Does such a substitution reduce consumer welfare? Is this effect mitigated by the presence of energy storage? Finally, when the storage capacity is binding, can this negative effect be reserved through a reasonable expansion of that capacity?

To answer the first question, we consider any energy mix $e \in \mathcal{E}$ and simultaneously increase renewable production capacity by ΔR while decreasing dispatchable energy availability by $\Delta D = -\left(\int_0^1 \varepsilon(t)\right) \Delta R$. In this way, the total available energy remains constant over $[0,1]$ and the resulting change in consumer welfare, for any $\forall e \in \mathcal{E}$, is given by:

$$\begin{aligned}\tfrac{\Delta W}{\Delta R} &= -\left(\int_0^1 \varepsilon(t)\right) u'\left(\tilde{c}_1(e)\right) + \int_0^1 u'\left(\min\left\{\max\left\{\tilde{c}_1(e), \varepsilon(t)R\right\}, \tilde{c}_2(e)\right\}\right) \varepsilon(t)dt \\ &= \int_0^1 \left(u'\left(\min\left\{\max\left\{\tilde{c}_1(e), \varepsilon(t)R\right\}, \tilde{c}_2(e)\right\}\right) - u'\left(\tilde{c}_1(e)\right)\right) \varepsilon(t)dt \end{aligned} \tag{50}$$

Now recall that $\tilde{c}_1(e) \leq \tilde{c}_2(e)$ which implies $\min\left\{\max\left\{\tilde{c}_1(e), \varepsilon(t)R\right\}, \tilde{c}_2(e)\right\} \geq \tilde{c}_1(e)$. Since the marginal utility u' is decreasing, we can say that:

$$\tfrac{\Delta W}{\Delta R} = \int_0^1 \underbrace{\left(u'\left(\min\left\{\max\left\{\tilde{c}_1(e), \varepsilon(t)R\right\}, \tilde{c}_2(e)\right\}\right) - u'\left(\tilde{c}_1(e)\right)\right)}_{\leq 0} \varepsilon(t)dt \leq 0 \tag{51}$$

We can therefore conclude that, for any energy mix $e \in \mathcal{E}$, replacing dispatchable energy with additional renewable capacity while maintaining total energy generation constant reduces consumer welfare. This result mainly stems from the fact that such a substitution increases the variability of the optimal consumption path. Let us now recall that $\partial_R W(e) \geq 0$. Thus, if the goal is to preserve consumer welfare, this result also suggests that the decrease in the availability of dispatchable energy must be less than the total increase in renewable production. In this case, from the consumers' perspective, the total increase in energy production over the cycle compensates for the additional fluctuations induced by renewable generation.

We can also ask whether the presence of active storage mitigates these welfare losses. In other words, it is interesting to compare the welfare losses of an energy mix, $e \in \mathcal{E}^b \cap \mathcal{E}^c$, to those of a hypothetical case in which storage is not allowed. In the latter case, the optimal electricity allocation is similar to that described Section 4, except that it now applies to an energy mix $e \in \mathcal{E}^b \cap \mathcal{E}^c$. Moreover, as can be seen in Fig.1 and in either Fig.2 or 3, the single plateau without storage, $c^{ns}(e)$, is lower than $\tilde{c}_1(e)$ in both cases. This follows from the fact that, in the no-storage case (see Fig.1), the yellow area corresponds solely to the amount of dispatchable energy. By contrast, when storage is available (see Fig.2 or 3), this area corresponds to the sum of dispatchable and stored energy (see Appendix H for a proof). Let us now denote the welfare losses when storage is not allowed by $\frac{\Delta W^{ns}}{\Delta R}$. This quantity is given by Eq.(50), where $\tilde{c}_1(e) = c^{ns}(e)$ and $\tilde{c}_2(e) = \varepsilon_M R$. We can therefore construct the difference in welfare losses for each e in $\mathcal{E}^b \cap \mathcal{E}^c$. After some manipulations (see Appendix H), we obtain:

$$\frac{\Delta W^{ns}}{\Delta R} - \frac{\Delta W}{\Delta R} = \int_0^1 \left(u'\left(\max\left\{\min\left\{\tilde{c}_1(e), \varepsilon(t)R\right\}, c^{ns}(e)\right\}\right) - u'\left(c^{ns}(e)\right)\right)\varepsilon(t)\,dt$$
$$+ \int_0^1 \left(u'\left(\max\left\{\tilde{c}_2(e), \varepsilon(t)R\right\}\right) - u'\left(\tilde{c}_2(e)\right)\right)\varepsilon(t)\,dt \tag{52}$$

Now recall from our previous discussion that $\tilde{c}_1(e) \geq c^{ns}(e)$, so that:

$$\max\left\{\min\left\{\tilde{c}_1(e), \varepsilon(t)R\right\}, c^{ns}(e)\right\} \geq c^{ns}(e) \tag{53}$$

Moreover, we observe that $\max\left\{\tilde{c}_2(e), \varepsilon(t)R\right\} \geq \tilde{c}_2(e)$. Since $u' < 0$, both integrals in Eq.(52) are negative, leading to $\frac{\Delta W^{ns}}{\Delta R} - \frac{\Delta W}{\Delta R} \leq 0$. Given that both terms are negative, we can conclude that the welfare losses when storage is not allowed, $\left|\frac{\Delta W^{ns}}{\Delta R}\right|$, are greater than the welfare losses with storage, $\left|\frac{\Delta W}{\Delta R}\right|$. Thus, we can say that active storage facilitates the energy transition from the consumer's perspective.

Finally, recall from Eq.(50) that any increase in storage capacity has a positive effect on welfare as long as this capacity remains binding (i.e., when $\frac{\sigma^-}{\sigma^+} > MRS_{\tilde{c}_1/\tilde{c}_2}$). This implies that the negative impact of substituting dispatchable energy with renewable energy can be offset by a simultaneous increase in storage capacity. To illustrate this point, consider again a situation in which renewable capacity increases by ΔR, while dispatchable capacity decreases by $\left(\int_0^1 \varepsilon(t)\,dt\right)\Delta R$. We can now couple this change with an expansion of storage capacity proportional to the change in renewable energy production over the cycle $[0,1]$, given by $k\left(\int_0^1 \varepsilon(t)\,dt\right)\Delta R$. In this case, the resulting variation in consumer welfare is:

$$\frac{\Delta W}{\Delta R} = -u'\left(\tilde{c}_1(e)\right)\left(\int_0^1 \varepsilon(t)\,dt\right) + \int_0^1 u'\left(\min\left\{\max\left\{\tilde{c}_1(e), \varepsilon(t)R\right\}, \tilde{c}_2(e)\right\}\right)\varepsilon(t)\,dt$$
$$+ k\sigma^+ u'\left(\tilde{c}_1(e)\right)\left(\frac{\sigma^-}{\sigma^+} - MRS_{\tilde{c}_1/\tilde{c}_2}\right)\left(\int_0^1 \varepsilon(t)\,dt\right) \tag{54}$$

Since the sum of the first two terms is negative while the last term is positive due to the binding storage capacity, we can easily identify the proportional expansion of storage capacity that offsets the initial negative welfare effect. This proportional factor k is given by:

$$k = \frac{u'(\tilde{c}_1(e))\left(\int_0^1 \varepsilon(t)\,dt\right) - \int_0^1 u'(\min\{\max\{\tilde{c}_1(e), \varepsilon(t)R\}, \tilde{c}_2(e)\})\varepsilon(t)\,dt}{\sigma^+ u'(\tilde{c}_1(e))\left(\frac{\sigma^-}{\sigma^+} - MRS_{\tilde{c}_1/\tilde{c}_2}\right)\left(\int_0^1 \varepsilon(t)\,dt\right)} > 0 \tag{55}$$

But this clearly raises the question of whether the magnitude of this factor is reasonable. To address this issue, let us observe that $\min\left\{\max\left\{\tilde{c}_1(e), \varepsilon(t)R\right\}, \tilde{c}_2(e)\right\} \leq \tilde{c}_2(e)$. It follows that:

$$k \leq \frac{(u'(\tilde{c}_1(e))-u'(\tilde{c}_2(e)))\left(\int_0^1 \varepsilon(t)\, dt\right)}{\sigma^+ u'(\tilde{c}_1(e))\left(\frac{\sigma^-}{\sigma^+}-MRS_{\tilde{c}_1/\tilde{c}_2}\right)\left(\int_0^1 \varepsilon(t)\, dt\right)}$$

$$= \frac{1}{\sigma^+}\frac{1-MRS_{\tilde{c}_1/\tilde{c}_2}}{\frac{\sigma^-}{\sigma^+}-MRS_{\tilde{c}_1/\tilde{c}_2}} < 1 \quad \text{since } \frac{\sigma^-}{\sigma^+} > 1 \text{ and } \sigma^+ > 1 \tag{56}$$

In conclusion, we can state that the negative welfare effect associated with substituting dispatchable energy sources with renewable energy sources can be offset by an increase in storage capacity, with the required expansion being less than the additional renewable production over the cycle.

The next proposition summarizes this discussion:

Proposition 5. *If renewable energy replaces dispatchable energy over the cycle while keeping total energy production constant, then:*
(i) consumer welfare decreases in the different scenarios that we have identified.
(ii) these welfare losses are nevertheless smaller when storage is activated during the renewable production cycle.
(iii) if storage capacity is limited, these welfare losses can be offset by adjusting the storage capacity through an increment that is lower than the increase in renewable energy production.

8. Further discussions

In this section, we discuss two extensions and a technical aspect of the model. We first examine the optimal choice of energy capacities and then characterize the pricing structure associated with the optimal consumption path. Finally, we explain why, in a framework focused on consumption, it is possible to restrict attention to a representative consumer without loss of generality.

8.1. The energy portfolio choice

In the analysis so far, we have abstracted from the costs associated with capacity installation and the production of dispatchable energy. In order to take these into consideration, we need to introduce explicit cost functions, C_D, C_R and C_S, corresponding respectively to dispatchable, renewable and storage capacities. Under the assumption of stationarity, these capacity costs primarily reflect operating expenses and the amortized capital repayment over the relevant time horizon. For dispatchable technologies, the cost function must also include fuel expenditures required to supply D units of energy over the cycle. This function, as in the seminal paper of Ambec and Crampes (2012), can be taken as linear. In this simple case, the optimization program can be written as:

$$\max_{(D,R,S)\,\in\,\mathcal{E}} W(D,R,S) - C_D(D) - C_R(R) - C_S(S) \tag{57}$$

where $W(D,R,S)$ is the welfare function obtained in Lemma 2 and C_i, $i = D,R,S$, are the cost functions associated with the capacity levels over the cycle. If these cost functions satisfy standard properties, particularly differentiability, then Lemma 2 implies that, for interior solutions, the marginal welfare of each energy source must be equal to its marginal cost. This generates a system of equations that can be solved to determine the optimal energy mix.

More sophisticated specifications, such as capacity accumulation, can also be introduced (see, for instance, Helm and Mier 2021 or Pommeret and Schubert 2022). Under capacity accumulation, the cost functions C_D, C_R, and C_S represent the investments undertaken in each period while total installed capacity becomes the cumulative sum of past investments, depreciated at different constant rates. This extension introduces an intertemporal trade-off in which the social planner balances the upfront investment costs against the consumer benefits generated by smoother consumption, enabled by the different production technologies.

In addition, we can allow for heterogeneous dispatchable generation technologies, such as coal, gas, or nuclear, each characterized by its own linear marginal cost. This naturally yields the classical merit-order structure: technologies with the lowest marginal cost are dispatched first up to their capacity constraint, followed sequentially by higher cost units (see, for instance, Léautier 2019). The framework can be further extended by introducing a merit order within renewable technologies (see Abrell et al. 2019), whereby the most productive units generate first. A similar merit order can also be applied to heterogeneous storage technologies (see Newbery 2018).

8.2. Electricity tariffs along the cycle

Throughout this work, we study the problem from the perspective of a social planner. As long as we introduce price taking behaviors, we can even associate a competitive equilibrium to this economy. Moreover, in the absence of externalities such as environmental damages, the Second Welfare Theorem implies that the electricity price at the competitive equilibrium is given by the marginal utility of the consumer along the optimal consumption path. This yields a tariff structure with two constant levels, a high and a low price, with real-time pricing between them.

When storage is present, the optimal consumption trajectory itself traces out the competitive tariff (see Fig.2 or 3). In the interval $[0, T_1]$, given that the marginal utility is decreasing in consumption, the price is constant at its upper level. In $[T_1, T_3]$, the consumer's flexibility allows consumption to adjust smoothly to the rising profile of renewable production, implying a decreasing price path up to a second plateau. In $[T_3, T_4]$, the price remains at its lower constant level, corresponding to the period of maximum renewable availability. In $[T_4, T_2]$, the price increases as renewable output falls, eventually reconnecting with the higher plateau. Finally, in $[T_2, 1]$, the price again remains constant at the high level. For the first regime, the tariff is straightforward with a unique off-peak price and real time price adjustment elsewhere (see Fig.1).

Also, a key modeling assumption in our analysis is that the consumer is perfectly flexible in her level of consumption. The tariff we derive therefore directly implements the planner's optimal consumption trajectory. This contrasts with the traditional Real-Time Pricing (RTP) literature, such as Borenstein and Holland (2005), which focuses on how consumers adjust their load in response to supply-driven price fluctuations. In their framework, peak demand is taken as exogenous and drives the shape of the optimal price path, making flat pricing inefficient compared to RTP. Our results show that even under full consumer flexibility, the efficient allocation exhibits consumption plateaus generated by renewable, dispatchable and stored energy, which translate into fixed prices similar to Time-of-Use (ToU) (see, for instance, Nicolson et al. 2018). Consequently, a full RTP schedule can be closely approximated by a ToU tariff augmented with a limited amount of real-time adjustment. The presence of storage capacities capable of storing excess renewable energy is what leads to this tariff structure.

8.3. Back to the representative consumer assumption

Now let us show how we can, without loss of generality, restrict our attention to a representative consumer. Suppose there are n heterogeneous consumers. Their utility function is given by $U_i(x_i(\cdot)) = \int_0^1 u_i(x_i(t))\,dt$, where each $u_i(x)$ is increasing, strictly concave and satisfies the usual Inada conditions. The social planner's problem can now be written as:

$$\max_{\left((x_i(t))_{i=1}^n, d(t), s^+(t), s^-(t)\right)} \sum_{i=1}^n \int_0^1 u_i(x_i(t))\,dt \tag{58}$$

This problem is subject to the same set of dynamic constraints given by Eq.(10), but with an additional constraint. This constraint stipulates that, for each t, distributed electricity corresponds to the amount of available energy due to a particular use of dispatchable and stored energy, i.e.,

$$\sum_{i=1}^n x_i(t) = \underbrace{d(t) - \sigma^+ s^+(t) + \sigma^- s^-(t) + \varepsilon(t)R}_{=X(t)} \tag{59}$$

Note that the allocation of dispatchable energy and the storage strategy are independent of how energy is distributed among consumers. This allows us to separate the questions of distribution and production, and consider the following problem:

$$V(X(\cdot)) = \max_{(x_i(\cdot))_{i=1}^n} \sum_{i=1}^n \int_0^1 u_i(x_i(t))\,dt \text{ s.t } \forall t,\ \sum_{i=1}^n x_i(t) = X(t) \tag{60}$$

The function $V(X(\cdot))$ stands for the aggregate indirect utility to be used in the production choice problem. In other words, it is the utility of our representative agent. We also observe that the previous problem is separable in time. This implies that we can restrict our attention to the sub-problem given by:

$$v(X) = \max_{(x_i)_{i=1}^n} \sum_{i=1}^n u_i(x_i) \text{ s.t } \sum_{i=1}^n x_i = X \tag{61}$$

and define the aggregate indirect utility as $V(X(\cdot)) = \int_0^1 v(X(t))\,dt$. So, if $v(X(t))$ shares the same restrictions as those we have imposed on our representative agent, both approaches lead to the same results. In other words, it remains to verify that $v(X(t))$ is increasing, strictly concave and satisfies the usual Inada conditions. This technical point is verified in Appendix I.

9. Concluding remarks

In this paper, our primary focus has been the investigation into the implications on consumer welfare of the substitution of dispatchable energy sources, mainly obtained from fossil fuels, by renewable energy sources that are frequently intermittent in nature. We started with two observations. On the one hand, the additional variability in supply requires consumers to be more flexible, which leads to welfare losses. Conversely, the development of storage capacity serves to mitigate these short-term fluctuations, thereby curtailing such welfare losses. To address this issue, we examined how intermittent renewable energy generation, dispatchable power supply and energy storage work together to determine the optimal electricity consumption path over a renewable production cycle. We then characterized the resulting level of consumer welfare. By solving this optimal control problem with fixed production and storage capacities, we identified three different regimes (no storage, abundant storage, and binding storage capacity) and their corresponding energy mixes. The effects on consumer welfare of replacing dispatchable energy with intermittent renewable energy sources were then discussed using these scenarios. As expected, consumer welfare declines with this substitution due to the need for additional flexibility. These welfare losses are smaller when storage is active, but they nevertheless persist. Finally, when storage capacity is limited, a moderate expansion of this capacity can fully offset these losses.

For a more comprehensive analysis, as suggested in Section 8.1, our model would require a deeper study of the composition of the electricity mix. In this paper, we considered only the optimal short-term adjustment of dispatchable, renewable, and storage capacities to supply variability. A more general framework would incorporate capacity accumulation, whereby new capacities result from investment decisions made in preceding cycles. This would introduce an intertemporal trade-off across the costs of different energy sources and yield an optimal time path for the energy mix, rather than the parametric analysis developed here. The resulting dynamics could then be compared with the existing literature (see, for instance, Helm and Mier 2021 or Pommeret and Schubert 2022).

It should also be noted that our discussion of electricity prices in Section 8.2 applies only when the conditions of the Second Welfare Theorem are satisfied. This excludes several types of market failure, such as price making decisions or environmental externalities which would both call for policy interventions. Regarding the latter, it is straightforward to introduce a damage function related to the production of dispatchable energy which is fossil-based. Such a damage function can be readily integrated into our optimization problem, as in Pommeret and Schubert (2022). However, addressing the policy issues induced by this externality would require explicitly modeling the price-taking behaviors of the different agents. To address market power, we even need to go a step further by identifying competitive and non-competitive behaviors (e.g., Ito and Reguant 2016, Acemoglu et al. 2017, Andrés-Cerezo and Fabra 2025). Both market failures could also be studied concomitantly as in García-Alaminos and Rubio (2021).

Finally, it is important to emphasize that the welfare losses highlighted in our model stem from an aversion to consumption flexibility. Consumers prefer a constant stream of consumption since the instantaneous utility does not depend explicitly on time but is defined only through the consumption path. If this condition does not hold, it may be possible to identify alternative consumption profiles, correlated or not, with the short-term renewable production cycle. For such analyzes, one may refer to Trotta (2020) or Reguant et al. (2025).

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Appendix A. Proof of Lemma 1

Points (i) and (ii). These follow directly from our discussion.

Point (iii). Assume that $\exists\,(t,t')$ for which $\forall t \in (t,t')$, $s^+(t)s^-(t) \neq 0$. In this case, $s^+(t) > 0$, $s^-(t) > 0$, and, according to the slackness conditions (see Eqs.(20) and (21)), we have $\mu_{s^+}(t) = \mu_{s^-}(t) = 0$. But if we sum Eq.(17) and Eq.(18), we get the following:

$$\lambda_S = \underbrace{(\sigma^- - \sigma^+)}_{<0}\underbrace{u'(x(t))}_{>0} < 0 \tag{A.1}$$

which contradicts the fact that λ_s is a non-negative constant (see Eq.(15)).

Point (iv). Now assume that $\exists\,(t_0,t_1)$ for which $\forall t \in (t_0,t_1)$, $s^+(t)d(t) \neq 0$. It follows from the slackness conditions given by Eqs.(19) and (20) that $\mu_d(t) = \mu_{s^+}(t) = 0$. Using the optimality conditions given by Eq.(16) and Eq.(17), we can therefore claim that:

$$-\sigma^+\lambda_D + \lambda_\Sigma - \lambda_S = 0 \tag{A.2}$$

Now, recall from Eq.(10) that all the stored energy is discharged during the cycle. As storage and discharge cannot occur simultaneously (see point (iii) of this proof), $\exists\,(t_2,t_3)$ for which $\forall t \in (t_2,t_3)$, $s^+(t) > 0$ and therefore $\mu_{s^-}(t) = 0$. From the optimality conditions given by Eq.(18) and Eq.(16) and the slackness condition given by Eq.(19), we obtain, respectively, $u'(x(t)) = \frac{\lambda_\Sigma}{\sigma^-}$ and $\frac{\lambda_\Sigma}{\sigma^-} - \lambda_D \leq 0$. As the co-states are constant on $[0,1]$ (see Eqs. (13) to (15)), we can say from Eq.(A.2) that:

$$\lambda_S = -\sigma^+\lambda_D + \lambda_\Sigma \leq -\sigma^+\lambda_D + \sigma^-\lambda_D < 0 \tag{A.3}$$

and contradicts the fact that λ_s is a non-negative constant (see Eq.(15)).

Appendix B. Proof of Proposition 1

Since the constraints of the optimization problem are linear, it remains to check that the Hamiltonian is concave with respect to the control and stock variables. This Hamiltonian

$$\mathcal{H}\left(d, s^+, s^-, \lambda_D, \lambda_\Sigma, \lambda_S, t\right) = u(d - \sigma^+ s^+ + \sigma^- s^- + \varepsilon(t)R) - \lambda_D d + \lambda_\Sigma \left(s^+ - s^-\right) - \lambda_S s^+ \tag{B.1}$$

is also independent of the stock variables $(D(t), \Sigma(t), S(t))$. We only have to verify that this function is concave with respect to the controls (d, s^+, s^-). A simple computation shows that its Hessian is given by:

$$\partial^2_{d,s^+,s^-}\mathcal{H} = u''(x(t)) \begin{bmatrix} 1 & -\sigma^+ & \sigma^- \\ -\sigma^+ & (\sigma^+)^2 & -\sigma^+\sigma^- \\ \sigma^- & -\sigma^+\sigma^- & (\sigma^-)^2 \end{bmatrix} = u''(x(t)) \begin{bmatrix} 1 \\ -\sigma^+ \\ \sigma^- \end{bmatrix} \begin{bmatrix} 1 & -\sigma^+ & \sigma^- \end{bmatrix} \tag{B.2}$$

It follows that:

$$\forall h \in \mathbb{R}^3,\ h^t \cdot \partial^2_{d,s^+,s^-}\mathcal{H} \cdot h = u''(x(t)) \left(h_1 - \sigma^+ h_2 + \sigma^- h_3\right)^2 \leq 0 \tag{B.3}$$

Appendix C. Proof of Proposition 2

Point (i). Let us show that:

$$\phi_1(c) = \int_0^1 \max\{c - \varepsilon(t)R, 0\}\, dt - D \tag{C.1}$$

admits a unique solution $\tilde{c}^a$ with the property that $\tilde{c}^a \geq \bar{x} = (u')^{-1}\left(\frac{\sigma^+}{\sigma^-} u'(\varepsilon_M R)\right)$. Obviously $\lim_{c\to\infty} \phi_1(c) = +\infty$ and, under the no storage condition (Eq.(26)), by construction $\lim_{c\to\bar{x}} \phi_1(c) \leq 0$. So, if $\phi_1(c)$ is increasing, there exists a unique $\tilde{c}^a \geq \bar{x}$ solution to $\phi_1(c) = 0$. To verify this point, divide the domain of $\phi_1(c)$ into two intervals: $[0, \varepsilon_M R)$ and $[\varepsilon_M R, +\infty)$. On the second, $c \geq \varepsilon_M R \geq \varepsilon(t)R$. It follows that:

$$\forall c \in [\varepsilon_M R, +\infty),\ \phi_1(c) = \int_0^1 (c - \varepsilon(t)R)\, dt - D = c - R\int_0^1 \varepsilon(t)dt - D \tag{C.2}$$

so that $\phi_1'(c) = 1$. Let us now move to the interval $[0, \varepsilon_M R)$. Since $c < \varepsilon_M R$, we can, by construction of the cycle $\varepsilon(t)$, assert that there exists, for each $c \in (0, \varepsilon_M R)$, a unique couple $(T_1(c), T_2(c))$ such that for $i = 1, 2$, $\varepsilon(T_i)R = c$. If T_M denotes the time of the peak of renewable production, we also have $0 < T_1(c) < T_M < T_2(c) < 1$. Moreover, since $\varepsilon(t)$ increases and decreases on, respectively, $(0, T_M)$ and $(T_M, 1)$, we can also say that $T_1'(c) > 0$ and $T_2'(c) < 0$. It follows that:

$$\forall c \in (0, \varepsilon_M R),\ \phi_1(c) = \int_0^{T_1(c)} (c - \varepsilon(t)R)\, dt + \int_{T_2(c)}^1 (c - \varepsilon(t)R)\, dt - D \tag{C.3}$$

and therefore:

$$\begin{aligned} \phi_1'(c) &= T_1(c) + T_1'(c)\,(c - \varepsilon(t)R)_{t=T_1(c)} + (1 - T_2(c)) - T_2'(c)\,(c - \varepsilon(t)R)_{t=T_2(c)} \\ &= 1 - T_2(c) + T_1(c) > 0 \text{ since } 0 < T_1(c) < T_2(c) < 1 \end{aligned} \tag{C.4}$$

Points (ii) and (iii). To save notation, we will omit the superscript a characterizing the case. So, let us now construct the consumption path $\forall t \in [0,1]$, $\tilde{x}(t) = \max\{\tilde{c}, \varepsilon(t)R\}$, the allocation of dispatchable $\forall t \in [0,1]$, $\tilde{d}(t) = \max\{\tilde{c} - \varepsilon(t)R, 0\}$ and the storage strategy $\forall t \in [0,1]$, $\tilde{s}^-(t) = \tilde{s}^+(t) = 0$. From Proposition 1, if constant co-states $(\tilde{\lambda}_D, \tilde{\lambda}_\Sigma, \tilde{\lambda}_S)$, and continuous and piecewise differentiable multipliers

$(\tilde{\mu}_d(t), \tilde{\mu}_{s^+}(t), \tilde{\mu}_{s^-}(t))$ exist and satisfy Eqs.(10), (15) and (16) to (21), then $\tilde{x}(t)$ is the solution to our electricity allocation problem.

Let us start with the isoperimetrical conditions given Eq.(10). By construction of $\tilde{c}$, we have $\int_0^1 \tilde{d}(t)dt = D$ and, since $\tilde{s}^+(t) = \tilde{s}^-(t) = 0$, we satisfy $\int_0^1 (s^+(t) - s^-(t))\, dt = 0$ and $\int_0^1 s^+(t)dt \leq S$. Lack of storage also implies that $\int_0^1 \tilde{s}^+(t)dt = 0 < S$ and therefore, by Eq.(15), that $\tilde{\lambda}_S = 0$. Since $\tilde{\lambda}_D$ is free (see Eq.(13)), we set $\tilde{\lambda}_D = u'(\tilde{c})$.

Let us now concentrate on Eqs.(16) and (19). Since the electricity consumption is given by $\tilde{x}(t) = \max\{\tilde{c}, \varepsilon(t)R\}$ and $\tilde{\lambda}_D = u'(\tilde{c})$, Eq.(16) provides the path of $\tilde{\mu}_d(t)$, given by:

$$\forall t \in [0,1]\,,\ \tilde{\mu}_d(t) = u'(\tilde{c}) - u'\left(\max\{\tilde{c}, \varepsilon(t)R\}\right) \tag{C.5}$$

Obviously, $\tilde{\mu}_d(t) \geq 0$ since $\tilde{c} \leq \max\{\tilde{c}, \varepsilon(t)R\}$ and $u''(x) < 0$. It remains to verify that $\forall t \in [0,1]$, $\tilde{d}(t)\tilde{\mu}_d(t) = 0$. Since $u''(x) < 0$, we can say that $\forall t \in [0,1]$

$$\tilde{d}(t) = \max\{\tilde{c} - \varepsilon(t)R, 0\} > 0 \Leftrightarrow \tilde{c} > \varepsilon(t)R \Leftrightarrow \tilde{\mu}_d(t) = u'(\tilde{c}) - u'\left(\max\{\tilde{c}, \varepsilon(t)R\}\right) = 0 \tag{C.6}$$

and, since $\tilde{d}(t) \geq 0$ and $\tilde{\mu}_d(t) \geq 0$, we also have $\tilde{\mu}_d(t) > 0 \Leftrightarrow \tilde{d}(t) = 0$. It follows that $\forall t \in [0,1]$, $\tilde{d}(t)\tilde{\mu}_d(t) = 0$.

Finally, let us concentrate on Eqs.(17), (18), (20) and (21). Since $\forall t \in [0,1]$, $\tilde{s}^+(t) = \tilde{s}^-(t) = 0$, the slackness conditions given by Eqs.(20) and (21) only require that $\forall t \in [0,1]$, $\tilde{\mu}_{s^+}(t), \tilde{\mu}_{s^-}(t) \geq 0$. If we now move to Eqs.(17) and (18), we get:

$$\forall t \in [0,1]\,,\ \begin{cases} \tilde{\mu}_{s^+}(t) = \sigma^+ u'(\tilde{x}(t)) - \lambda_\Sigma \\ \tilde{\mu}_{s^-}(t) = \lambda_\Sigma - \sigma^- u'(\tilde{x}(t)) \end{cases} \tag{C.7}$$

Since λ_Σ is free (see Eq.(14)), the non-negativity of $\tilde{\mu}_{s^+}(t)$ and $\tilde{\mu}_{s^-}(t)$ therefore requires that:

$$\forall t \in [0,1]\,,\ \lambda_\Sigma \in \left[\sigma^- u'(\tilde{x}(t)),\, \sigma^+ u'(\tilde{x}(t))\right] \tag{C.8}$$

In order to show that such a λ_Σ exists, first recall that no storage occurs if $\bar{x} \leq \tilde{x}(t) \Leftrightarrow u'(\bar{x}) \geq u'(\tilde{x}(t))$ since $u''(x) < 0$. Moreover, by Eq.(22), $\bar{x} = (u')^{-1}\left(\frac{\sigma^+}{\sigma^-} u'(\varepsilon_M R)\right)$. It follows that:

$$\max_{t \in [0,1]} \sigma^- u'(\tilde{x}(t)) \leq \sigma^- u'(\bar{x}) = \sigma^+ u'(\varepsilon_M R) \tag{C.9}$$

Since $\tilde{x}(t) = \max\{\tilde{c}, \varepsilon(t)R\} \leq \max\{\tilde{c}, \varepsilon_M R\}$ and $u''(x) < 0$, we can also say that:

$$\min_{t \in [0,1]} \sigma^+ u'(\tilde{x}(t)) \geq \sigma^+ u'\left(\max\{\tilde{c}, \varepsilon_M R\}\right) \tag{C.10}$$

Finally, since $u'(\varepsilon_M R) \leq u'\left(\max\{\tilde{c}, \varepsilon_M R\}\right)$, we can define a λ_Σ which satisfies Eq.(C.8) given, for instance, by:

$$\lambda_\Sigma = \tfrac{\sigma^+}{2}\left(u'(\varepsilon_M R) + u'\left(\max\{\tilde{c}, \varepsilon_M R\}\right)\right) \tag{C.11}$$

Appendix D. Proof of Proposition 3

Point (i). We proceed in two steps. First, we construct the threshold $\underline{x}$ and exhibit its property. In a second step, we show that there exists a unique $c_1 \in [\underline{x}\,, \bar{x})$. satisfying Eq.(35).

Let us first study $\underline{x}$ which solves $\int_0^1 s^+(f(\underline{x}), t) - S = 0$. By Eqs.(33) and (34), we know that $f(x)$ is a bijection from $(0, +\infty)$ into $(0, +\infty)$. So, let us concentrate, by the definition of $s^+(y,t)$ (see Eq.(31)), on:

$$\varphi(y) = \tfrac{1}{\sigma^+} \int_0^1 \max\{\varepsilon(t)R - y, 0\}\, dt - S \tag{D.1}$$

If there exists a unique $\underline{y} > 0$ solution to $\varphi(y) = 0$, then $\underline{x} = f^{-1}(\underline{y})$ exists and is unique. Let us observe that (i) $\forall y \geq \varepsilon_M R$, $\varphi(y) = -S$ and (ii) by our assumption (see Eq.(7)), $\lim_{x\to 0} \varphi(y) = \frac{R}{\sigma^+} \int_0^1 \varepsilon(t)dt - S > 0$. It therefore remains to verify that $\varphi(y)$ is strictly decreasing on $(0, \varepsilon_M R)$. From the property of our cycle $\varepsilon(t)$ on $[0,1]$, we can construct, for each $y \in (0, \varepsilon_M R)$, a unique couple of times $T_3(y) < T_M < T_4(y)$ with the property that for $i = 3,4$, $\varepsilon(T_i)R = y$. It follows that $\varphi(y)$ becomes $\varphi(y) = \frac{1}{\sigma^+} \int_{T_3(y)}^{T_4(y)} (\varepsilon(t)R - y)\, dt - S$ and :

$$\varphi'(y) = \frac{1}{\sigma^+}(-\underbrace{(\varepsilon(t)R - y)|_{t=T_3(x)}}_{=0} T_3'(x) + \underbrace{(\varepsilon(t)R - y)|_{t=T_4(x)}}_{=0} T_4'(x) - (T_4(x) - T_3(x))\, y) < 0 \tag{D.2}$$

Let us now show that there exists a unique $\tilde{c}_1^b \in [\ \underline{x}\ , \bar{x})$ satisfying Eq.(35). So, define:

$$\phi_2(c) = \int_0^1 \max\{c - \varepsilon(t)R, 0\}\, dt - \sigma^- \int_0^1 \tfrac{1}{\sigma^+} \max\{\varepsilon(t)R - f(c), 0\}\, dt - D \tag{D.3}$$

From Eqs.(C.1) and (D.1), we observe that $\phi_2(c) = \phi_1(c) - \sigma^-(\varphi(f(c)) + S)$. Since, by Eq.(22), $f(\bar{x}) = \varepsilon_M R$ and $f'(c) > 0$, we can say, by Eq.(D.1), that $\forall c \geq \bar{x}$, $\varphi(f(c)) = -S$. It follows, by the condition given by Eq.(28), that:

$$\forall c \geq \bar{x},\ \phi_2(c) = \int_0^1 \max\{c - \varepsilon(t)R, 0\}\, dt - D \geq \int_0^1 \max\{\bar{x} - \varepsilon(t)R, 0\}\, dt - D > 0 \tag{D.4}$$

Now let us consider $c < \underline{x}$. Since $f'(c) > 0$, we can state, by construction of φ (see Eq.(D.1)), that $\forall c < \underline{x}$, $\varphi(f(c)) \geq \varphi(f(\underline{x}) = 0$. Moreover, the condition given by Eq.(37) implies that $\forall c < \underline{x}$, $\phi_1(c) < \sigma^- S$ and it follows that $\forall c < \underline{\mathrm{x}}$, $\phi_2(c) < \sigma^- S - \sigma^- S = 0$. We can therefore claim that there exists $\tilde{c}_1^b$ solution to $\phi_2(c) = 0$ with the property that $\tilde{c}_1^b \in [\ \underline{\mathrm{x}}, \bar{x})$. Finally, from Eqs.(34),(C.4) and (D.2), we can say $\forall c \in (\underline{\mathrm{x}}, \bar{x})$, $\phi_2'(c) > 0$. This ensures uniqueness.

Points (ii) to (iv). We again neglect the superscript b of the case and show that our solution candidates satisfy the sufficient optimality conditions of Proposition 1. So, let us set $\tilde{x}(t) = \min\{\max\{\tilde{c}_1, \varepsilon(t)R\}, \tilde{c}_2)\}$ and $\tilde{s}^+(t) = \frac{1}{\sigma^+} \max\{\varepsilon(t)R - \tilde{c}_2), 0\}$ with $\tilde{c}_2 = f(\tilde{c}_2)$ given by Eq.(33). The choice of $\tilde{d}(t)$ and $\tilde{s}^-(t)$ (see (iv) of Proposition 3) is less obvious due to multiple solutions. To solve this problem, we consider any continuous and positive selections $\tilde{d}(t)$ and $\tilde{s}^-(t)$ that verify the conditions given in (iv) of Proposition 3.[7]

The isoperimetrical constraints given by Eq.(10) are obviously satisfied. $\tilde{d}(t)$ and $\tilde{s}^-(t)$ verify their respective condition by construction of these selections. Moreover, since $\tilde{c}_1 \geq \underline{x}$, the storage capacity is not binding so that $\int_0^1 \tilde{s}^+(t) \leq S$.

Now let us set $\tilde{\lambda}_D = u'(\tilde{c}_1)$, $\tilde{\lambda}_\Sigma = \sigma^+ u'(\tilde{c}_2))$ and $\tilde{\lambda}_S = 0$ (so that condition (15) is satisfied). If we now concentrate on the Lagrangian multipliers, we immediately obtain from Eqs (16) to (18) that:

$$\forall t \in [0,1],\ \tilde{\mu}_d(t) = u'(\tilde{c}_1) - u'(\tilde{x}(t)) \tag{D.5}$$

$$\forall t \in [0,1],\ \tilde{\mu}_{s^+}(t) = -\sigma^+ u'(\tilde{c}_2) + \sigma^+ u'(\tilde{x}(t)) \tag{D.6}$$

$$\forall t \in [0,1],\ \tilde{\mu}_{s^-}(t) = \sigma^+ u'(\tilde{c}_2) - \sigma^- u'(\tilde{x}(t)) \tag{D.7}$$

It remains to show that the slackness conditions given by Eqs.(19) to (21) hold.

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[7] Readers may wonder whether such selections exist. The example given by:

$$\tilde{d}(t) = \frac{D}{D + \sigma^- \int_0^1 \tilde{s}^+(t)dt} \max\{\tilde{c}_1 - \varepsilon(t)R, 0\} \text{ and } \tilde{s}^-(t) = \frac{\int_0^1 \tilde{s}^+(t)dt}{D + \sigma^- \int_0^1 \tilde{s}^+(t)dt} \max\{\tilde{c}_1 - \varepsilon(t)R, 0\}$$

satisfy the conditions given in (iv) of Proposition 3.

Concerning the non-negativity of these multipliers, recall first that $\tilde{x}(t) = \min\{\max\{\tilde{c}_1, \varepsilon(t)R\}, \tilde{c}_2\}$ and that $\tilde{c}_1 < \tilde{c}_2$. This implies that (i) $\tilde{x}(t) = \tilde{c}_1$ if $\varepsilon(t)R \leq \tilde{c}_1$ and (ii) $\tilde{x}(t) = \min\{\varepsilon(t)R, \tilde{c}_2\}$, else. In any case $\tilde{x}(t), \geq \tilde{c}_1$ and since $u''(x) < 0$, $u'(\tilde{x}(t)) \leq u'(\tilde{c}_1)$. It follows from Eq.(D.5) that:

$$\forall t \in [0,1], \ \tilde{\mu}_d(t) = \begin{cases} 0 \text{ if } \varepsilon(t)R \leq \tilde{c}_1 \\ u'(\tilde{c}_1) - u'(\tilde{x}(t)), \text{ else} \end{cases} \geq 0 \tag{D.8}$$

Since $\tilde{c}_1 < \tilde{c}_2$, we can also deduce, from the definition $\tilde{x}(t)$, that (i) $\tilde{x}(t) = \tilde{c}_2$ if $\varepsilon(t)R \geq \tilde{c}_2$ and (ii) $\tilde{x}(t) < \tilde{c}_2$, else. As $u''(x) < 0$, we get $u'(\tilde{x}(t)) \geq u'(\tilde{c}_2))$. Hence, Eq.(D.6) implies that:

$$\forall t \in [0,1], \ \tilde{\mu}_{s^+}(t) = \begin{cases} 0 \text{ if } \varepsilon(t)R \geq \tilde{c}_2 \\ \sigma^+ (u'(\tilde{x}(t)) - u'(\tilde{c}_2), \text{ else} \end{cases} \geq 0 \tag{D.9}$$

Finally, since $\tilde{c}_2 = (u')^{-1}\left(\frac{\sigma_-}{\sigma_+}u'(\tilde{c}_1)\right)$, Eq.(D.7) becomes:

$$\forall t \in [0,1], \ \tilde{\mu}_{s^-}(t) = \sigma^- u'(\tilde{c}_1) - \sigma^- u'(\tilde{x}(t)) = \sigma^- \tilde{\mu}_d(t) \geq 0 \tag{D.10}$$

It remains to verify the three exclusion conditions. First,recall that the selections, $\tilde{d}(t)$ and $\tilde{s}^-(t)$, verify:

$$\forall t \in [0,1], \ d(t) + \sigma^- s^-(t) = \max\{\tilde{c}_1 - \varepsilon(t)R, 0\} \tag{D.11}$$

Since $\tilde{d}(t), \tilde{s}^-(t) \geq 0$, we can therefore claim that $\tilde{d}(t) = \tilde{s}^-(t) = 0$ if $\varepsilon(t)R \geq \tilde{c}_1$. It follows respectively by Eq.(D.8) and Eq.(D.10) that $\forall t \in [0,1]$, $\tilde{\mu}_d(t)\tilde{d}(t) = 0$ and $\tilde{\mu}_{s^-}(t)\tilde{s}^-(t) = 0$. Finally, since $\tilde{s}^+(t) = \frac{1}{\sigma^+}\max\{\varepsilon(t)R - \tilde{c}_2, 0\}$, we can say that $\tilde{s}^+(t) = 0$ if $\tilde{c}_2 \geq \varepsilon(t)R$. Hence, by Eq.(D.9), $\forall t \in [0,1]$, $\tilde{\mu}_{s^+}(t)\tilde{s}^+(t) = 0$.

Appendix E. Proof of Proposition 4

Point (i). Let us start with the existence and uniqueness of $\tilde{c}_2^c$. From Eq.(41), $\tilde{c}_2^c$ solves $\varphi(\tilde{c}_2^c) = 0$ as defined in Eq.(D.1) of Appendix D. By using the first step of point (i) of the latter Appendix, we immediately conclude that $\tilde{c}_2^c = f(\underline{x})$ with $f(x)$ given by Eq.(33).

Now observe that $\tilde{c}_1^c$ solves $\phi_3(c) = 0$ with

$$\phi_3(c) = \int_0^1 \max\{c - \varepsilon(t)R, 0\}\, dt - \sigma^- S - D = \phi_1(c) - \left(D + \sigma^- S\right) \tag{E.1}$$

From (i) of Appendix C, we know that $\phi_1'(c) > 0$; the same therefore holds for $\phi_3(c)$. Moreover $\phi_3(0) = -(D + \sigma^- S) < 0$ and, with the condition given by Eq.(43), $\phi_3(\underline{x}) > 0$. It follows that there exists a unique $\tilde{c}_1^c < \underline{x}$ solving this equation.

Points (ii) to (iv). To spare notation, we omit the superscript c of the case under consideration and, as in Appendix D, we set $\tilde{x}(t) = \min\{\max\{\tilde{c}_1, \varepsilon(t)R\}, \tilde{c}_2\}$ and $\tilde{s}^+(t) = \frac{1}{\sigma^+}\max\{\varepsilon(t)R - \tilde{c}_2, 0\}$ and select two continuous functions, $\tilde{d}(t)$ and $\tilde{s}^-(t)$, that satisfy $\forall t \in [0,1]$, $\tilde{d}(t) + \sigma^- \tilde{s}^-(t) = \max\{\tilde{c}_1 - \varepsilon(t)R, 0\}$, $\int_0^1 \tilde{d}(t)dt = D$ but now with $\int_0^1 \tilde{s}^-(t)dt = S$. It follows that the isoperimetrical constraints given by Eq.(10) are again satisfied, except that the storage constraint is now binding.

Since $\tilde{\lambda}_D$ and $\tilde{\lambda}_\Sigma$ are free (see Eqs.(13) and (14)), we set $\tilde{\lambda}_D = u'(\tilde{c}_1)$ and $\tilde{\lambda}_\Sigma = \sigma^- u'(\tilde{c}_1)$. We now fix $\tilde{\lambda}_S = \sigma^- u'(\tilde{c}_1) - \sigma^- u'(\tilde{c}_2)$. By Eq.(40), we know that $\tilde{\lambda}_S \geq 0$ and, because the storage constraint is binding, $S(1) = 0$. It follows that Eq.(15) is satisfied.

It follows, by Eqs.(16) to (18), that the Lagrangian multipliers are given by:

$$\tilde{\mu}_d(t) = u'(\tilde{c}_1) - u'(\tilde{x}(t)) \tag{E.2}$$
$$\tilde{\mu}_{s^+}(t) = \sigma^+ \left(u'(\tilde{x}(t)) - u'(\tilde{c}_2)\right) \tag{E.3}$$
$$\tilde{\mu}_{s^-}(t) = \sigma^- \left(u'(\tilde{c}_1) - u'(\tilde{x}(t))\right) = \sigma^- \tilde{\mu}_d(t) \tag{E.4}$$

Concerning $\tilde{\mu}_d(t)$, we can, since $\tilde{c}_1 < \tilde{c}_2$, apply the same argument as in Appendix D and obtain that:

$$\forall t \in [0,1]\,,\; \tilde{\mu}_d(t) = \begin{cases} 0 \text{ if } \varepsilon(t)R \leq \tilde{c}_1 \\ u'(\tilde{c}_1) - u'(\tilde{x}(t))\,, \text{ else} \end{cases} \geq 0 \tag{E.5}$$

From Eq.(E.4), we immediately observe that $\tilde{\mu}_{s^-}(t) \geq 0$. Finally, concerning $\tilde{\mu}_{s^+}(t)$, let us recall that $\tilde{x}(t) = \min\{\max\{\tilde{c}_1, \varepsilon(t)R\}, \tilde{c}_2\}$ and that $\tilde{c}_1 < \tilde{c}_2$. This implies that (i) $\tilde{x}(t) = \max\{\tilde{c}_1, \varepsilon(t)R\}$ if $\varepsilon(t)R \leq \tilde{c}_2$ and (ii) $\tilde{x}(t) = \tilde{c}_2$ if $\varepsilon(t)R > \tilde{c}_2$. In any case, $\tilde{x}(t) \leq \tilde{c}_2$, or $u'(\tilde{x}(t)) \geq u'(\tilde{c}_2)$ since $u''(x) < 0$. It follows, from Eq.(E.3), that:

$$\forall t \in [0,1]\,,\; \tilde{\mu}_{s^+}(t) = \begin{cases} \sigma^+\left(u'(\tilde{x}(t)) - u'(\tilde{c}_2)\right. \text{ if } \varepsilon(t)R < \tilde{c}_2 \\ 0, \text{ else} \end{cases} \geq 0 \tag{E.6}$$

It remains to verify the three exclusions conditions contained in Eqs.(19) to (21) hold. With the same argument as in Appendix D, we conclude that $\forall t \in [0,1]\,,\; \tilde{\mu}_d(t)\tilde{d}(t) = 0$ and $\tilde{\mu}_{s^-}(t)\tilde{s}^-(t) = 0$. Finally, since $\tilde{s}^+(t) = \frac{1}{\sigma^+}\max\{\varepsilon(t)R - \tilde{c}_2, 0\}$, we can say that $\tilde{s}^+(t) = 0$ if $\tilde{c}_2 \geq \varepsilon(t)R$. Hence, by Eq.(E.6), $\forall t \in [0,1]\,,\; \tilde{\mu}_{s^+}(t)\tilde{s}^+(t) = 0$.

Appendix F. Consumption plateaus and comparative statics

Case (a): Without storage.

The derivative of Eq.(27) with respect to D is given by:[8]

$$\int_0^{\tilde{T}_1^a} \partial_D \tilde{c}^a dt + \underbrace{(\tilde{c}^a - \varepsilon(t)R)|_{t=\tilde{T}_1^a}}_{=0} \partial_D \tilde{T}_1^a + \int_{\tilde{T}_2^a}^1 \partial_D \tilde{c}^a dt - \underbrace{(\tilde{c}^a - \varepsilon(t)R)|_{t=\tilde{T}_2^a}}_{=0} \partial_D \tilde{T}_1^a - 1 = 0 \tag{F.1}$$

and from the definition of $\tilde{T}_i^a$, we know that $(\tilde{c}^a - \varepsilon(t)R)|_{t=\tilde{T}_1^a} = (\tilde{c}^a - \varepsilon(t)R)|_{t=\tilde{T}_2^a} = 0$. Moreover, $\partial_D \tilde{c}^a$ is independent of t and $\tilde{T}_2^a < 1$. It follows that:

$$\partial_D \tilde{c}^a \left(1 + \tilde{T}_1^a - \tilde{T}_2^a\right) - 1 = 0 \Leftrightarrow \partial_D \tilde{c}^a = \left(1 + \tilde{T}_1^a - \tilde{T}_2^a\right)^{-1} > 0 \tag{F.2}$$

Using a similar argument, the derivative of Eq.(27) with respect to R becomes:

$$\int_0^{\tilde{T}_1^a} \left(\partial_R \tilde{c}^a - \varepsilon(t)\right) dt + \int_{\tilde{T}_2^a}^1 \left(\partial_R \tilde{c}^a - \varepsilon(t)\right) dt = 0 \tag{F.3}$$

We deduce that:

$$\partial_R \tilde{c}^a = \left(1 + \tilde{T}_1^a - \tilde{T}_2^a\right)^{-1} \left(\int_0^{\tilde{T}_1^a} \varepsilon(t) dt + \int_{\tilde{T}_2^a}^1 \varepsilon(t) dt\right) > 0 \tag{F.4}$$

Case (b): With abundant storage.

First, let us substitute $\tilde{c}_2^b$ in Eq.(38) by its value given by Eq.(39). Using the definition of switching times $\tilde{T}_i^b$ $i = 1, \ldots, 4$, the derivative of this new identity with respect to D is:

$$\int_0^{\tilde{T}_1^b} \partial_D \tilde{c}_1^b dt + \underbrace{\left(\tilde{c}_1^b - \varepsilon(t)R\right)\big|_{t=\tilde{T}_1^b}}_{=0} \partial_D \tilde{T}_1^b + \int_{\tilde{T}_2^b}^1 \partial_D \tilde{c}_1^b dt - \underbrace{\left(\tilde{c}_1^b - \varepsilon(t)R\right)\big|_{t=\tilde{T}_2^b}}_{=0} \partial_D \tilde{T}_2^b$$

$$+ \frac{\sigma^-}{\sigma^+} \int_{\tilde{T}_3^b}^{\tilde{T}_4^b} f'(\tilde{c}_1^b) \partial_D \tilde{c}_1^b dt - \underbrace{\left(f(\tilde{c}_1^b) - \varepsilon(t)R\right)\big|_{t=\tilde{T}_3^b}}_{=0} \partial_D \tilde{T}_3^b + \underbrace{\left(f(\tilde{c}_1^b) - \varepsilon(t)R\right)\big|_{t=\tilde{T}_4^b}}_{=0} \partial_D \tilde{T}_4^b - 1 = 0 \tag{F.5}$$

[8]To simplify notations, we omit in what follow, the argument (D, R) of the different functions unless it is necessary for a better understanding.

Since $\tilde{c}_1^b$ and $\partial_D \tilde{c}_1^b$ are time-independent, we obtain:

$$\left(\tilde{T}_1^b + 1 - \tilde{T}_2^b + \tfrac{\sigma^-}{\sigma^+}(\tilde{T}_3^b - \tilde{T}_4^b) f'(\tilde{c}_1^b)\right) \partial_D \tilde{c}_1^b - 1 = 0 \tag{F.6}$$

Now observe that $f'(\tilde{c}_1^b)$ is given by Eq.(34) and recall that $u'' < 0$, $\tilde{T}_2^b < 1$ and $\tilde{T}_3^b < \tilde{T}_4^b$. It follows that:

$$\partial_D \tilde{c}_1^b = \frac{u''(\tilde{c}_2^b)}{(1+\tilde{T}_1^b-\tilde{T}_2^b)u''(\tilde{c}_2^b)+\left(\frac{\sigma^-}{\sigma^+}\right)^2(\tilde{T}_4^b-\tilde{T}_3^b)u''(\tilde{c}_1^b)} > 0 \tag{F.7}$$

We now compute the derivative of Eq.(38) with respect to R. Using the definition of the switching times $\tilde{T}_i^b$ $i = 1, \dots, 4$ and the fact that $\tilde{c}_1^b$ and $\partial_D \tilde{c}_1^b$ are time-independent, we get that:

$$\left(\tilde{T}_1^b + 1 - \tilde{T}_2^b + \tfrac{\sigma^-}{\sigma^+}(\tilde{T}_4^b - \tilde{T}_3^b) f'(\tilde{c}_1^b)\right) \partial_R \tilde{c}_1^b - \int_0^{\tilde{T}_1^b} \varepsilon(t)dt - \int_{\tilde{T}_2^b}^1 \varepsilon(t)dt - \frac{\sigma^-}{\sigma^+}\int_{\tilde{T}_3^b}^{\tilde{T}_4^b} \varepsilon(t)dt = 0 \tag{F.8}$$

Since $u'' < 0$ and $\tilde{T}_2^b < 1$, $\tilde{T}_3^b < \tilde{T}_4^b$, we can conclude that:

$$\partial_R \tilde{c}_1^b = \frac{\left(\int_0^{\tilde{T}_1^b} \varepsilon(t)dt + \int_{\tilde{T}_2^b}^1 \varepsilon(t)d + \frac{\sigma^-}{\sigma^+}\int_{\tilde{T}_3^b}^{\tilde{T}_4^b} \varepsilon(t)dt\right)u''(\tilde{c}_2^b)}{(1+\tilde{T}_1^b-\tilde{T}_2^b)u''(\tilde{c}_2^b)+\left(\frac{\sigma^-}{\sigma^+}\right)^2(\tilde{T}_4^b-\tilde{T}_3^b)u''(\tilde{c}_1^b)} > 0 \tag{F.9}$$

To obtain the derivatives of $\tilde{c}_2^b$ with respect to D and R, we simply use Eq.(39). By the chain rule, we get $\partial_D \tilde{c}_2^b = f'(\tilde{c}_1^b)\partial_D \tilde{c}_1^b$ and $\partial_R \tilde{c}_2^b = f'(\tilde{c}_1^b)\partial_R \tilde{c}_1^b$. Since $u'' < 0$ and $\tilde{T}_2^b < 1$, $\tilde{T}_3^b < \tilde{T}_4^b$, it follows that:

$$\partial_D \tilde{c}_2^b = \frac{\frac{\sigma^-}{\sigma^+}u''(\tilde{c}_1^b)}{(1+\tilde{T}_1^b-\tilde{T}_2^b)u''(\tilde{c}_2^b)+\left(\frac{\sigma^-}{\sigma^+}\right)^2(\tilde{T}_4^b-\tilde{T}_3^b)u''(\tilde{c}_1^b)} > 0 \tag{F.10}$$

and

$$\partial_R \tilde{c}_1^b = \frac{\left(\int_0^{\tilde{T}_1^b} \varepsilon(t)dt + \int_{\tilde{T}_2^b}^1 \varepsilon(t)d + \frac{\sigma^-}{\sigma^+}\int_{\tilde{T}_3^b}^{\tilde{T}_4^b} \varepsilon(t)dt\right)\frac{\sigma^-}{\sigma^+}u''(\tilde{c}_1^b)}{(1+\tilde{T}_1^b-\tilde{T}_2^b)u''(\tilde{c}_2^b)+\left(\frac{\sigma^-}{\sigma^+}\right)^2(\tilde{T}_4^b-\tilde{T}_3^b)u''(\tilde{c}_1^b)} > 0 \tag{F.11}$$

Case (c): With binding storage.

Since $\tilde{c}_2^c$ is not in Eq.(45) and vice-versa for $\tilde{c}_1^c$ and Eq.(46), we can work identity per identity. So if we compute the derivative of Eq.(45) with respect to D we get:

$$\int_0^{\tilde{T}_1^c} \partial_D \tilde{c}_1^c dt + \underbrace{(\tilde{c}_1^c - \varepsilon(t)R)|_{t=\tilde{T}_1^b}}_{=0} \partial_D \tilde{T}_1^c + \int_{\tilde{T}_2^c}^1 \partial_D \tilde{c}_1^c dt - \underbrace{(\tilde{c}_1^c - \varepsilon(t)R)|_{t=\tilde{T}_2^c}}_{=0} \partial_D \tilde{T}_2^c - 1 = 0 \tag{F.12}$$

Since $\partial_D \tilde{c}_1^c$ is independent of t and $\tilde{T}_2^c < 1$, we can say that:

$$\partial_D \tilde{c}_1^c = \frac{1}{(\tilde{T}_1^c+1-\tilde{T}_2^c)} > 0 \tag{F.13}$$

Now by differentiating with respect to R, we have:

$$\int_0^{\tilde{T}_1^c} \left(\partial_R \tilde{c}_1^c - \varepsilon(t)\right) dt + \int_{\tilde{T}_2^c}^1 \left(\partial_R \tilde{c}_1^c - \varepsilon(t)\right) dt = 0 \Rightarrow \partial_R \tilde{c}_1^c = \frac{\int_0^{\tilde{T}_1^c} \varepsilon(t)dt + \int_{\tilde{T}_2^c}^1 \varepsilon(t)dt}{(\tilde{T}_1^c+1-\tilde{T}_2^c)} > 0 \tag{F.14}$$

Finally, working with S gives the following:

$$\int_0^{\tilde{T}_1^c} \partial_S \tilde{c}_1^c dt + \int_{\tilde{T}_2^c}^1 \partial_S \tilde{c}_1^c dt - \sigma^- = 0 \Rightarrow \partial_S \tilde{c}_1^c = \frac{\sigma^-}{(\tilde{T}_1^c+1-\tilde{T}_2^c)} > 0 \tag{F.15}$$

If we now move to Eq.(46), we immediately observe that $\partial_D \tilde{c}_2^c = 0$. Moreover, the derivative of Eq.(46) with respect to R is given by:

$$\int_{\tilde{T}_3^c}^{\tilde{T}_4^c} (\varepsilon(t) - \partial_R \tilde{c}_2^c)\, dt = 0 \Rightarrow \partial_R \tilde{c}_2^c = \frac{\int_{\tilde{T}_3^c}^{\tilde{T}_4^c} \varepsilon(t) dt}{\tilde{T}_4^c - \tilde{T}_3^c} > 0 \tag{F.16}$$

By doing the same computation with respect to S, we get:

$$\int_{\tilde{T}_3^c}^{\tilde{T}_4^c} (-\partial_S \tilde{c}_2^c)\, dt - \sigma^+ = 0 \Rightarrow \partial_S \tilde{c}_2^c = -\frac{\sigma^+}{\tilde{T}_4^c - \tilde{T}_3^c} < 0 \tag{F.17}$$

Appendix G. Proof of Lemma 2

(i) Preliminary remarks

From the conditions given by Eqs.(26), (28), (37) and (43), and the upper bound on dispatchable energy given by Eq.(8), we can say that the different energy mix domains corresponding to no-storage, abundant storage and binding storage are respectively given by:

$$\mathcal{E}^a = \left\{ e \in \mathcal{E} : \int_0^1 \max\{\bar{x}(e) - \varepsilon(t)R, 0\}\, dt \leq D \leq R \int_0^1 (\varepsilon_M - \varepsilon(t))\, dt \right\} \tag{G.1}$$

$$\mathcal{E}^b = \left\{ e \in \mathcal{E} : \int_0^1 \max\{\underline{x}(e) - \varepsilon(t)R, 0\}\, dt - \sigma^- S \leq D < \int_0^1 \max\{\bar{x}(e) - \varepsilon(t)R, 0\}\, dt \right\} \tag{G.2}$$

$$\mathcal{E}^c = \left\{ e \in \mathcal{E} : 0 \leq D < \int_0^1 \max\{\underline{x}(e) - \varepsilon(t)R, 0\}\, dt - \sigma^- S \right\} \tag{G.3}$$

with $\bar{x}(e) = f^{-1}(\varepsilon_M R) = (u')^{-1}\left(\frac{\sigma^+}{\sigma^-} u'(\varepsilon_M R)\right)$ (see Eq.(22)) and $\underline{x}$ solving $\int_0^1 s^+(f(\underline{x}(e)), t) - S = 0$ (see (i) of Appendix D).

In addition, note that, by construction, $\bar{x}(e)$ and $\underline{x}(e)$ are continuous functions, but neither of them takes D as an argument. This implies that the left and right terms of each inequality that define these domains are independent of D. Now recall that $\varepsilon_M R \geq \bar{x}(e) > \underline{x}(e) > 0$, it follows that:

$$R \int_0^1 (\varepsilon_M - \varepsilon(t))\, dt \;\geq \int_0^1 \max\{\bar{x}(e) - \varepsilon(t)R, 0\}\, dt \tag{G.4}$$

and

$$\int_0^1 \max\{\bar{x} - \varepsilon(t)R, 0\}\, dt > \int_0^1 \max\{\underline{x}(e) - \varepsilon(t)R, 0\}\, dt - \sigma^- S \tag{G.5}$$

Finally, by condition (43), we know that:

$$\int_0^1 \max\{\underline{x}(e) - \varepsilon(t)R, 0\}\, dt - \sigma^- S > 0 \tag{G.6}$$

These observations first ensure that $\mathcal{E}^a$, $\mathcal{E}^b$ and $\mathcal{E}^c$ form a partition of $\mathcal{E}$. But they also say that (i) $\mathcal{E}^a$ has boundary only with $\mathcal{E}^b$ given by $\int_0^1 \max\{\bar{x}(e) - \varepsilon(t)R, 0\}\, dt = D$, (ii) the boundary between $\mathcal{E}^b$ and $\mathcal{E}^c$ is given by $\int_0^1 \max\{\underline{x}(e) - \varepsilon(t)R, 0\}\, dt = D + \sigma^- S$ and (iii) $\mathcal{E}^a$ and $\mathcal{E}^c$ has no common boundary. This simplifies the proof of the continuity of $\tilde{c}_1(e)$ and $\tilde{c}_2^b(e)$ since we only have to look at "what happens" on each side of the common boundary.

(ii) Continuity of $\tilde{c}_1(e)$ and $\tilde{c}_2(e)$

The boundary $\int_0^1 \max\{\bar{x}(e) - \varepsilon(t)R, 0\}\, dt = D$ separates $\mathcal{E}^a$ from $\mathcal{E}^b$. So let us consider two sequences $e_n^a \to e$ and $e_n^b \to e$ that verify $e_n^a \in \mathcal{E}^a$, $e_n^b \in \mathcal{E}^b$ and e is a point of this boundary. Let us show that

$\left(\tilde{c}_1\left(e_n^a\right), \tilde{c}_2\left(e_n^a\right)\right)_{n\in\mathbb{N}}$ and $\left(\tilde{c}_1\left(e_n^b\right), \tilde{c}_2\left(e_n^b\right)\right)_{n\in\mathbb{N}}$ converge to the same vector $\left(\tilde{c}_1\left(e\right)\tilde{c}_2\left(e\right)\right)$. Let us start with the sequence $\left(\tilde{c}_1\left(e_n^a\right), \tilde{c}_2\left(e_n^a\right)\right)_{n\in\mathbb{N}}$. By construction, $\forall e \in \mathcal{E}^a$, $\tilde{c}_2\left(e^a\right) = \varepsilon_M R$. Hence, $\tilde{c}_2\left(e_n^a\right) \to \tilde{c}_2\left(e\right) = \varepsilon_M R$. Concerning $\tilde{c}_1\left(e_n^a\right)$, let us first observe by Eq.(C.1) that $\phi_1(\bar{x}(e)) = 0$ so that $\tilde{c}_1\left(e\right) = \bar{x}(e)$. Moreover, $\forall n$, $\tilde{c}_1\left(e_n^a\right)$ solves $\phi_1(\tilde{c}_1\left(e_n^a\right)) = 0$. Since ϕ_1 is continuous and admits a unique zero (see (i) of Appendix C), we can say that $\tilde{c}_1\left(e_n^a\right) \to \tilde{c}_1\left(e\right) = \bar{x}(e)$. Let us now move to the sequence $\left(\tilde{c}_1\left(e_n^b\right), \tilde{c}_2\left(e_n^b\right)\right)_{n\in\mathbb{N}}$. From Eq.(D.3), we know that $\forall n$, $\phi_2(\tilde{c}_1\left(e_n^b\right)) = 0$. Hence, by continuity of ϕ_2, we have $\phi_2(\tilde{c}_1\left(e\right)) = 0$. Since e belongs to the boundary, we can say that:

$$\phi_2(\tilde{c}_1\left(e\right)) = -\sigma^- \int_0^1 \tfrac{1}{\sigma^+} \max\left\{\varepsilon(t)R - f(\tilde{c}_1\left(e\right)), 0\right\} dt = 0 \tag{G.7}$$

It follows that $\tilde{c}_1^b\left(e\right) \geq f^{-1}\left(\varepsilon_M R\right) = \bar{x}(e)$. But we also know that (see (i) of Proposition 3), $\forall n$, $\tilde{c}_1\left(e_n^b\right) \leq \bar{x}(e_n^b) = f^{-1}(\varepsilon_M R_n)$. Hence, by pushing to the limit, continuity implies that $\tilde{c}_1\left(e\right) \leq \bar{x}(e) = f^{-1}(\varepsilon_M R)$. In other words, we can conclude that $\tilde{c}_1\left(e_n^b\right) \to \tilde{c}_1\left(e\right) = \bar{x}(e)$. Concerning $\tilde{c}_2\left(e_n^b\right)$, let us recall, by (i) of Proposition 3, that $\forall n$, $\tilde{c}_2\left(e_n^b\right) = f\left(\tilde{c}_1\left(e_n^b\right)\right)$. As $\tilde{c}_1\left(e_n^b\right) \to \bar{x}(e) = f^{-1}(\varepsilon_M R)$, it follows that $\tilde{c}_2\left(e_n^b\right) \to \varepsilon_M R$.

Let now move to the boundary $\int_0^1 \max\left\{\underline{x}(e) - \varepsilon(t)R, 0\right\} dt = \sigma^- S + D$ that separates $\mathcal{E}^b$ from $\mathcal{E}^c$ and let us consider two sequences $e_n^b \to e$ and $e_n^c \to e$ that verify $e_n^b \in \mathcal{E}^b$, $e_n^c \in \mathcal{E}^c$ and e is a point of this boundary. We start by studying $\left(\tilde{c}_1\left(e_n^b\right), \tilde{c}_2\left(e_n^b\right)\right)_{n\in\mathbb{N}}$. So let us observe, by Eq.(D.1), that $\varphi\left(f(\underline{x}(e)\right) = 0$. By Eq.(D.3) and the definition of the boundary under consideration, we can claim that:

$$\phi_2(\underline{x}(e)) = \int_0^1 \max\left\{\underline{x}(e) - \varepsilon(t)R, 0\right\} dt - \sigma^- \left(\varphi(f(\ \underline{x}(e))) + S\right) = 0 \tag{G.8}$$

It follows that $\tilde{c}_2\left(e\right) = \underline{x}(e)$. Uniqueness of the solution (see (i) of Appendix D) and continuity of ϕ_2 ensure that $\tilde{c}_1\left(e_n^b\right) \to \tilde{c}_1\left(e\right) = \underline{x}(e)$. Moreover, by continuity of $f(c_1)$, we get that $\tilde{c}_2\left(e_n^b\right) = f\left(\tilde{c}_1\left(e_n^b\right)\right) \to f\left(\underline{x}(e)\right)$. Now, let us concentrate on $\left(\tilde{c}_1\left(e_n^c\right), \tilde{c}_2\left(e_n^c\right)\right)_{n\in\mathbb{N}}$. From Eq.(E.1) , we know that $\forall n$, $\phi_3(\tilde{c}_1\left(e_n^c\right)) = 0$. By continuity of ϕ_3, this implies that, at the limit, $\tilde{c}_1\left(e\right)$ solves:

$$\phi_3(\tilde{c}_1\left(e\right)) = \int_0^1 \max\left\{\tilde{c}_1^c\left(e\right) - \varepsilon(t)R, 0\right\} dt - \sigma^- S - D = 0 \tag{G.9}$$

As the solution to this equation is unique (see (i) of Appendix E), we deduce, from the definition of the boundary under consideration that $\tilde{c}_1\left(e\right) = \underline{x}(e)$. Moreover, by continuity of ϕ_3 and f, $\tilde{c}_1\left(e_n^c\right) \to \underline{x}(e)$. Finally, recall from (i) of Proposition 4 that $\tilde{c}_2^c\left(e_n^c\right) = f\left(\underline{x}(e_n^c)\right)$. Hence, by continuity, $\tilde{c}_2^c\left(e_n^c\right) \to \tilde{c}_2^c\left(e\right) = f\left(\underline{x}(e)\right)$.

At that point, we can conclude that $\tilde{c}_1\left(x\right)$ and $\tilde{c}_2\left(x\right)$ are continuous functions. As the consumption path writes $\tilde{x}\left(t, e\right) = \min\left\{\max\left\{\tilde{c}_1(e), \varepsilon(t)R\right\}, \tilde{c}_2(e)\right\}$, this one is also a continuous function both in t and e and the same holds for the consumer welfare $W(e) = \int_0^1 u\left(\tilde{x}\left(t, e\right)\right) dt$.

(iii) The welfare function $W(e)$

Now we calculate the gradient of $W(e)$. We begin by studying this gradient in each of the three cases and verify, in a final step, that this one is globally continuous. The notation, particularly concerning switching times, is the same as in Appendix F. We omit the arguments of the various functions in order to lighten the notation.

Case (a): Without storage

In this case, $W^a = \left(\tilde{T}_1^a + 1 - \tilde{T}_2^a\right) u\left(\tilde{c}^a\right) + \int_{\tilde{T}_1^a}^{\tilde{T}_2^a} u\left(\varepsilon(t)R\right) dt$. Taking the derivative with respect to D, we get the following.

$$\begin{aligned}
\partial_D W^a &= \left(\partial_D \tilde{T}_1^a - \partial_D \tilde{T}_2^a\right) u(\tilde{c}_1^a) + \left(\tilde{T}_1^a + 1 - \tilde{T}_2^a\right) u'(\tilde{c}_1^a)\partial_D \tilde{c}_1^a - \partial_D \tilde{T}_1^a u\left(\varepsilon(\tilde{T}_1^a)R\right) + \partial_D \tilde{T}_2^a u\left(\varepsilon(\tilde{T}_2)R\right) \\
&= \left(\tilde{T}_1^a + 1 - \tilde{T}_2^a\right) u'(\tilde{c}_1^a)\partial_D \tilde{c}_1^a \text{ (since } \varepsilon(\tilde{T}_i^a)R = \tilde{c}_1^a \text{ for } i = 1, 2) \\
&= u'(\tilde{c}_1^a) \text{ (see } \partial_D \tilde{c}_1^a \text{ in Tab.1)}
\end{aligned} \tag{G.10}$$

Working now with R, we obtain:

$$\begin{aligned}\partial_R W^a &= \left(\partial_R \tilde{T}_1^a - \partial_R \tilde{T}_2^a\right) u(\tilde{c}_1^a) + \left(\tilde{T}_1^a + 1 - \tilde{T}_2^a\right) u'(\tilde{c}_1^a)\partial_R \tilde{c}^a + \int_{\tilde{T}_1^a}^{\tilde{T}_2^a} u'(\varepsilon(t)R)\varepsilon(t)dt \\ &\quad -\partial_R \tilde{T}_1^a u(\varepsilon(\tilde{T}_1^a)R) + \partial_R \tilde{T}_2^a u'(\varepsilon(\tilde{T}_2^a)R) \\ &= \left(1 + \tilde{T}_1^a - \tilde{T}_2^a\right) u'(\tilde{c}_1^a)\partial_R \tilde{c}_1^a + \int_{\tilde{T}_1^a}^{\tilde{T}_2^a} u'(\varepsilon(t)R)\varepsilon(t)dt \text{ (since } \varepsilon(\tilde{T}_1^a)R = \varepsilon(\tilde{T}_2^a)R = \tilde{c}_1^a) \\ &= u'(\tilde{c}_1^a)\left(\int_0^{\tilde{T}_1^a} \varepsilon(t)dt + \int_{\tilde{T}_2^a}^1 \varepsilon(t)dt\right) + \int_{\tilde{T}_1^a}^{\tilde{T}_2^a} u'(\varepsilon(t)R)\varepsilon(t)dt \text{ (see } \partial_D \tilde{c}_1^a \text{ in Tab.1)} \\ &= \int_0^1 u'\left(\max\left\{\tilde{c}_1^a, \varepsilon(t)R\right\}\right)\varepsilon(t)dt \end{aligned} \tag{G.11}$$

Finally, by construction, $\partial_S W^a = 0$.

Case (b): With abundant storage

In this case, $W^b = \left(\tilde{T}_1^b + 1 - \tilde{T}_2^b\right) u\left(\tilde{c}_1^b\right) + \left(T_4^b - T_3^b\right) u\left(\tilde{c}_2^b\right) + \int_{\tilde{T}_1^b}^{\tilde{T}_3^b} u\left(\varepsilon(t)R\right) dt + \int_{\tilde{T}_4^b}^{\tilde{T}_2^b} u\left(\varepsilon(t)R\right) dt$. Taking the derivative with respect to D, we get the following:

$$\begin{aligned}\partial_D W^b &= \left(\partial_D \tilde{T}_1^b - \partial_D \tilde{T}_2^b\right) u\left(\tilde{c}_1^b\right) + \left(\tilde{T}_1^b + 1 - \tilde{T}_2^b\right) u'\left(\tilde{c}_1^b\right)\partial_D \tilde{c}_1^b + \left(\partial_D T_4^b - \partial_D T_3^B\right) u\left(\tilde{c}_2^b\right) \\ &\quad + \left(T_4^b - T_3^b\right) u'\left(\tilde{c}_2^b\right)\partial_D \tilde{c}_2^b - \partial_D \tilde{T}_1^b u\left(\varepsilon(\tilde{T}_1^b)R\right) + \partial_D \tilde{T}_3^b u\left(\varepsilon(\tilde{T}_3^b)R\right) \\ &\quad -\partial_D \tilde{T}_4^b u\left(\varepsilon(\tilde{T}_4)R\right) + \partial_D \tilde{T}_2^b u\left(\varepsilon(\tilde{T}_2^b)R\right) \end{aligned} \tag{G.12}$$

Now note, by construction, that $\varepsilon(\tilde{T}_i^b)R = \tilde{c}_1^b$ for $i = 1, 2$ and $\varepsilon(\tilde{T}_i^b)R = \tilde{c}_2^b$ for $i = 3, 4$. It follows that:

$$\begin{aligned}\partial_D W^b &= \left(\tilde{T}_1^b + 1 - \tilde{T}_2^b\right) u'\left(\tilde{c}_1^b\right)\partial_D \tilde{c}_1^b + \left(T_4^b - T_3^b\right) u'\left(\tilde{c}_2^b\right)\partial_D \tilde{c}_2^b \\ &= u'\left(\tilde{c}_1^b\right) \frac{\left(\tilde{T}_1^b + 1 - \tilde{T}_2^b\right) u''(\tilde{c}_2^b) + \left(T_4^b - T_3^b\right)\frac{\sigma^-}{\sigma^+} u''(\tilde{c}_1^b)\frac{u'\left(\tilde{c}_2^b\right)}{u'\left(\tilde{c}_3^b\right)}}{\left(1 + \tilde{T}_1^b - \tilde{T}_2^b\right) u''(\tilde{c}_2^b) + \left(\frac{\sigma^-}{\sigma^+}\right)^2 (\tilde{T}_4^b - \tilde{T}_3^b) u''(\tilde{c}_1^b)} \text{ (see } \partial_D \tilde{c}_1^b,\ \partial_D \tilde{c}_1^b \text{ in Tab.2)} \\ &= u'\left(\tilde{c}_1^b\right) \ \left(\text{since } \frac{u'\left(\tilde{c}_2^b\right)}{u'\left(\tilde{c}_1^b\right)} = \frac{\sigma^-}{\sigma^+}\right) \end{aligned} \tag{G.13}$$

Taking the derivative with respect to R, we get the following.

$$\begin{aligned}\partial_R W^b &= \left(\partial_R \tilde{T}_1^b - \partial_R \tilde{T}_2^b\right) u\left(\tilde{c}_1^b\right) + \left(\tilde{T}_1^b + 1 - \tilde{T}_2^b\right) u'\left(\tilde{c}_1^b\right)\partial_R \tilde{c}_1^b + \left(\partial_R T_4^b - \partial_R T_3^b\right) u\left(\tilde{c}_2^b\right) \\ &\quad + \left(T_4^b - T_3^b\right) u'\left(\tilde{c}_2^b\right)\partial_R \tilde{c}_2^b + \int_{\tilde{T}_1^b}^{\tilde{T}_3^b} u'\left(\varepsilon(t)R\right)\varepsilon(t)dt - \partial_R \tilde{T}_1^b u\left(\varepsilon(\tilde{T}_1^b)R\right) \\ &\quad +\partial_R \tilde{T}_3^b u\left(\varepsilon(\tilde{T}_3^b)R\right) + \int_{\tilde{T}_4^b}^{\tilde{T}_2^b} u'\left(\varepsilon(t)R\right)\varepsilon(t)dt - \partial_R \tilde{T}_4^b u\left(\varepsilon(\tilde{T}_4^b)R\right) + \partial_R \tilde{T}_2^b u\left(\varepsilon(\tilde{T}_2^b)R\right) \end{aligned} \tag{G.14}$$

As $\varepsilon(\tilde{T}_i^b)R = \tilde{c}_1^b$ for $i = 1, 2$ and $\varepsilon(\tilde{T}_i^b)R = \tilde{c}_2^b$ for $i = 3, 4$, we obtain the following:

$$\begin{aligned}\partial_R W^b &= \underbrace{\left(\tilde{T}_1^b + 1 - \tilde{T}_2^b\right) u'\left(\tilde{c}_1^b\right)\partial_R \tilde{c}_1^b + \left(T_4^b - T_3^b\right) u'\left(\tilde{c}_2^b\right)\partial_R \tilde{c}_2^b}_{=A} \\ &\quad + \int_{\tilde{T}_1^b}^{\tilde{T}_3^b} u'\left(\varepsilon(t)R\right)\varepsilon(t)dt + \int_{\tilde{T}_4^b}^{\tilde{T}_2^b} u'\left(\varepsilon(t)R\right)\varepsilon(t)dt \end{aligned} \tag{G.15}$$

If we now use the results of Tab.2 and the fact that $\frac{u'\left(\tilde{c}_2^b\right)}{u'\left(\tilde{c}_1^b\right)} = \frac{\sigma^-}{\sigma^+}$, A becomes:

$$A = \frac{\left(\int_0^{\tilde{T}_1^b} \varepsilon(t)dt + \int_{\tilde{T}_2^b}^1 \varepsilon(t)d + \frac{\sigma^-}{\sigma^+}\int_{\tilde{T}_3^b}^{\tilde{T}_4^b} \varepsilon(t)dt\right) u'\left(\tilde{c}_1^b\right)\left(u''(\tilde{c}_2^b)(\tilde{T}_1^b+1-\tilde{T}_2^b)+\frac{\sigma^-}{\sigma^+}u''(\tilde{c}_1^b)\left(T_4^b-T_3^b\right)\frac{u'\left(\tilde{c}_2^b\right)}{u'\left(\tilde{c}_1^b\right)}\right)}{(1+\tilde{T}_1^b-\tilde{T}_2^b)u''(\tilde{c}_2^b)+\left(\frac{\sigma^-}{\sigma^+}\right)^2(\tilde{T}_4^b-\tilde{T}_3^b)u''(\tilde{c}_1^b)}$$

$$= \left(\int_0^{\tilde{T}_1^b} \varepsilon(t)dt + \int_{\tilde{T}_2^b}^1 \varepsilon(t)dt + \frac{\sigma^-}{\sigma^+}\int_{\tilde{T}_3^b}^{\tilde{T}_4^b} \varepsilon(t)dt\right) u'\left(\tilde{c}_1^b\right)$$

$$= \int_0^{\tilde{T}_1^b} u'\left(\tilde{c}_1^b\right)\varepsilon(t)dt + \int_{\tilde{T}_2^b}^1 u'\left(\tilde{c}_1^b\right)\varepsilon(t)d + \int_{\tilde{T}_3^b}^{\tilde{T}_4^b} u'\left(\tilde{c}_2^b\right)\varepsilon(t)dt \tag{G.16}$$

We can therefore say that:

$$\begin{aligned}\partial_R W^b &= \int_0^{\tilde{T}_1^b} u'\left(\tilde{c}_1^b\right)\varepsilon(t)dt + \int_{\tilde{T}_2^b}^1 u'\left(\tilde{c}_1^b\right)\varepsilon(t)d + \int_{\tilde{T}_3^b}^{\tilde{T}_4^b} u'\left(\tilde{c}_2^b\right)\varepsilon(t)dt \\ &\quad + \int_{\tilde{T}_1^b}^{\tilde{T}_3^b} u'\left(\varepsilon(t)R\right)\varepsilon(t)dt + \int_{\tilde{T}_4^b}^{\tilde{T}_2^b} u'\left(\varepsilon(t)R\right)\varepsilon(t)dt \\ &= \int_0^1 u'\left(\min\left\{\max\left\{\tilde{c}_1^b, \varepsilon(t)R\right\}, \tilde{c}_2^b\right\}\right)\varepsilon(t)dt \quad (\text{since } \tilde{c}_1^b \le \tilde{c}_2^b)\end{aligned} \tag{G.17}$$

Finally, by construction, $\partial_S W^b = 0$.

<u>Case (c):</u> With binding storage

Like in the previous case, $W^c = \left(\tilde{T}_1^c + 1 - \tilde{T}_2^c\right) u\left(\tilde{c}_1^c\right) + \left(T_4^c - T_3^c\right) u\left(\tilde{c}_2^c\right) + \int_{\tilde{T}_1^c}^{\tilde{T}_3^c} u\left(\varepsilon(t)R\right)dt + \int_{\tilde{T}_4^c}^{\tilde{T}_2^c} u\left(\varepsilon(t)R\right)dt$. So if we take the derivative with respect to D, we obtain the same result as in Eq.(G.13). The same simplifications apply since $\varepsilon(\tilde{T}_i^c)R = \tilde{c}_1^c$ for $i = 1, 2$ and $\varepsilon(\tilde{T}_i^c)R = \tilde{c}_2^c$ for $i = 3, 4$. But $\partial_D \tilde{c}_1^c$ and $\partial_D \tilde{c}_2^c$ are now taken from Tab.3. It follows that:

$$\partial_D W^c = \left(\tilde{T}_1^c + 1 - \tilde{T}_2^c\right) u'\left(\tilde{c}_1^c\right)\partial_D \tilde{c}_1^c + \left(T_4^c - T_3^c\right) u'\left(\tilde{c}_2^c\right)\partial_D \tilde{c}_2^c = u'\left(\tilde{c}_1^c\right) \tag{G.18}$$

If we now move to the derivative with respect to R, the same simplifications apply as in Eq.(G.15) and we get:

$$\begin{aligned}\partial_R W^c &= \left(\tilde{T}_1^c + 1 - \tilde{T}_2^c\right) u'\left(\tilde{c}_1^c\right)\partial_R \tilde{c}_1^c + \left(T_4^c - T_3^c\right) u'\left(\tilde{c}_2^c\right)\partial_R \tilde{c}_2^c \\ &\quad + \int_{\tilde{T}_1^c}^{\tilde{T}_3^c} u'\left(\varepsilon(t)R\right)\varepsilon(t)dt + \int_{\tilde{T}_4^c}^{\tilde{T}_2^c} u'\left(\varepsilon(t)R\right)\varepsilon(t)dt\end{aligned} \tag{G.19}$$

By using now the results of Tab.3, we observe that:

$$\begin{aligned}\partial_R W^c &= \int_0^{\tilde{T}_1^c} u'\left(\tilde{c}_1^c\right)\varepsilon(t)dt + \int_{\tilde{T}_2^c}^1 u'\left(\tilde{c}_1^c\right)\varepsilon(t)dt + \int_{\tilde{T}_3^c}^{\tilde{T}_4^c} u'\left(\tilde{c}_2^c\right)\varepsilon(t)dt \\ &\quad + \int_{\tilde{T}_1^c}^{\tilde{T}_3^c} u'\left(\varepsilon(t)R\right)\varepsilon(t)dt + \int_{\tilde{T}_4^c}^{\tilde{T}_2^c} u'\left(\varepsilon(t)R\right)\varepsilon(t)dt \\ &= \int_0^1 u'\left(\min\left\{\max\left\{\tilde{c}_1^c, \varepsilon(t)R\right\}, \tilde{c}_2^c\right\}\right)\varepsilon(t)dt \quad (\text{since } \tilde{c}_1^b \le \tilde{c}_2^b)\end{aligned} \tag{G.20}$$

Finally, if we move to the derivative with respect to S, we obtain, by applying the same simplifications that:

$$\partial_S W^c = \left(\tilde{T}_1^c + 1 - \tilde{T}_2^c\right) u'\left(\tilde{c}_1^c\right)\partial_S \tilde{c}_1^c + \left(T_4^c - T_3^c\right) u'\left(\tilde{c}_2^c\right)\partial_S \tilde{c}_2^c \tag{G.21}$$

Using again Tab.3, we can say that:

$$\partial_S W^c = \sigma^- u'\left(\tilde{c}_1^c\right) - \sigma^+ u'\left(\tilde{c}_2^c\right) = u'\left(\tilde{c}_1^c\right)\sigma^+\left(\frac{\sigma^-}{\sigma^+} - \frac{u'\left(\tilde{c}_2^c\right)}{u'\left(\tilde{c}_1^c\right)}\right) \tag{G.22}$$

Pooling together these results

By Eqs.(G.10), (G.13) and (G.18), $\partial_D V = u'\left(\tilde{c}_1\right)$. Moreover, by our extension of $\tilde{c}_2$ to case (a), we can say, by Eqs.(G.11), (G.17) and (G.20), that $\partial_R W = \int_0^1 u'\left(\min\left\{\max\left\{\tilde{c}_1, \varepsilon(t)R\right\}, \tilde{c}_2\right\}\right)\varepsilon(t)dt$. By Eqs. (22), (32) and (40), we know that in case (a) $\frac{\sigma^-}{\sigma^+} < \frac{u'(\varepsilon_M R)}{u'\left(\tilde{c}_1^a\right)}$, in case (b) $\frac{\sigma^-}{\sigma^+} = \frac{u'\left(\tilde{b}_2^c\right)}{u'\left(\tilde{b}_1^c\right)}$ and in case (c) $\frac{\sigma^-}{\sigma^+} > \frac{u'\left(\tilde{c}_2^c\right)}{u'\left(\tilde{c}_1^c\right)}$. We can therefore say that $\partial_S W = \max\left\{u'\left(\tilde{c}_1\right)\sigma^+\left(\frac{\sigma^-}{\sigma^+} - \frac{u'(\tilde{c}_2)}{u'(\tilde{c}_1)}\right), 0\right\}$. Finally, we know that $\tilde{c}_1$ and $\tilde{c}_2$ are continuous functions, meaning that the gradient of W is also continuous.

Appendix H. Proof of Proposition 5

The proofs of points (i) and (iii) directly follow from our discussion while, for point (ii), it remains to show that $c^{ns}(e) \leq \tilde{c}_1(e)$ and that Eq.(52) holds.

(i) $c^{ns}(e) \leq \tilde{c}_1(e)$

Let us first recall the definition of $c^{ns}(e)$. We consider an energy mix $e \in \mathcal{E}^b \cup \mathcal{E}^c$ and assume that storage is not allowed. This consumption plateau therefore solves $\phi_1(c) = \int_0^1 \max\left\{c - \varepsilon(t)R, 0\right\}dt - D = 0$. From our previous results in (i) of Appendix C and the fact that $\lim_{c\to 0}\phi_1(c) = -D$, we can conclude that $c^{ns}(e)$ is well-defined for each $e \in \mathcal{E}^b \cup \mathcal{E}^c$.

Now let us recall, respectively, from (i) of Appendix D and Appendix E that $\tilde{c}_1(e)$ either solves $\phi_2(c) = \phi_1(c) - \sigma^-\left(\varphi(f(c)) + S\right) = 0$ or $\phi_3(c) = \phi_1(c) - \sigma^- S = 0$. Now observe that $\phi_2(c^{ns}(e)) = -\sigma^-\left(\varphi(f(c^{ns}(e))) + S\right) \leq 0$ and $\phi_3(c^{ns}(e)) = -\sigma^- S \leq 0$. As both $\phi_2(c)$ and $\phi_3(c)$ are increasing in c, we can conclude that $\forall e \in \mathcal{E}^b \cup \mathcal{E}^c$, $c^{ns}(e) \leq \tilde{c}_1(e)$.

(ii) The construction of Eq.(52)

As usual, let us first introduce the following switching times $T_i^{ns}(e) = \varepsilon^{-1}\left(\frac{c^{ns}(e)}{R}\right)$, $i = 1, 2$, $\tilde{T}_i(e) = \varepsilon^{-1}\left(\frac{\tilde{c}_1(e)}{R}\right)$, $i = 1, 2$ and $\tilde{T}_i^b(e) = \varepsilon^{-1}\left(\frac{\tilde{c}_2(e)}{R}\right)$, $i = 3, 4$. From (i) and since we know that $\tilde{c}_1(e) < \tilde{c}_2(e)$, we have that, $T_1^{ns}(e) < \tilde{T}_1(e) < \tilde{T}_3(e) < \tilde{T}_4(e) < \tilde{T}_2(e) < T_2^{ns}(e)$.

Let us now move to the computation of Eq.(52). By neglecting e in the later notation, we know, by Eq.(50), that:

$$\begin{aligned}
\frac{\Delta W^{ns}}{\Delta R} &= -u'\left(c^{ns}\right)\int_0^1 \varepsilon(t)dt + \int_0^{T_1^{ns}} u'\left(c^{ns}\right)\varepsilon(t)dt + \int_{T_1^{ns}}^{T_2^{ns}} u'\left(\varepsilon(t)R\right)\varepsilon(t)dt + \int_{T_2^{ns}}^1 u'\left(c^{ns}\right)\varepsilon(t)dt \\
&= -u'\left(c^{ns}\right)\int_{T_1^{ns}}^{T_2^{ns}} \varepsilon(t)dt + \int_{T_1^{ns}}^{T_2^{ns}} u'\left(\varepsilon(t)R\right)\varepsilon(t)dt
\end{aligned} \tag{H.1}$$

and

$$\begin{aligned}
\frac{\Delta W}{\Delta R} &= -u'\left(\tilde{c}_1\right)\int_0^1 \varepsilon(t)dt + \int_0^{\tilde{T}_1} u'\left(\tilde{c}_1\right)\varepsilon(t)dt + \int_{\tilde{T}_1}^{\tilde{T}_3} u'\left(\varepsilon(t)R\right)\varepsilon(t)dt + \int_{\tilde{T}_3}^{\tilde{T}_4} u'\left(\tilde{c}_2\right)\varepsilon(t)dt \\
&\quad + \int_{\tilde{T}_4}^{\tilde{T}_2} u'\left(\varepsilon(t)R\right)\varepsilon(t)dt + \int_{\tilde{T}_2}^1 u'\left(\tilde{c}_1\right)\varepsilon(t)dt \\
&= -u'\left(\tilde{c}_1\right)\int_{\tilde{T}_1}^{\tilde{T}_2} \varepsilon(t)dt + \int_{\tilde{T}_1}^{\tilde{T}_3} u'\left(\varepsilon(t)R\right)\varepsilon(t)dt + \int_{\tilde{T}_3}^{\tilde{T}_4} u'\left(\tilde{c}_2\right)\varepsilon(t)dt + \int_{\tilde{T}_4}^{\tilde{T}_2} u'\left(\varepsilon(t)R\right)\varepsilon(t)dt
\end{aligned} \tag{H.2}$$

By computing the difference, we get, after reorganization and simplification, that:

$$\begin{aligned}\tfrac{\Delta W^{ns}}{\Delta R} - \tfrac{\Delta W}{\Delta R} = & \int_{T_1^{ns}}^{\tilde{T}_1} \left(u'\left(\varepsilon(t)R\right) - u'\left(c^{ns}\right)\right)\varepsilon(t)dt + \left(u'\left(\tilde{c}_1\right) - u'\left(c^{ns}\right)\right)\int_{\tilde{T}_1}^{\tilde{T}_2} \varepsilon(t)dt \\ & + \int_{\tilde{T}_2}^{T_2^{ns}} \left(u'\left(\varepsilon(t)R\right) - u'\left(c^{ns}\right)\right)\varepsilon(t)dt + \int_{\tilde{T}_3}^{\tilde{T}_4} \left(u'\left(\varepsilon(t)R\right) - u'\left(\tilde{c}_2\right)\right)\varepsilon(t)dt \end{aligned} \tag{H.3}$$

By grouping together the first three terms (recall that $c^{ns} \leq \tilde{c}_1$) and using the definition of $\tilde{c}_2$, we finally get that:

$$\begin{aligned}\tfrac{\Delta W^{ns}}{\Delta R} - \tfrac{\Delta W}{\Delta R} = & \int_0^1 \left(u'\left(\max\left\{\min\left\{\tilde{c}_1, \varepsilon(t)R\right\}, c^{ns}\right\}\right) - u'\left(c^{ns}\right)\right)\varepsilon(t)dt \\ & + \int_0^1 \left(u'\left(\max\left\{\tilde{c}_2, \varepsilon(t)R\right\}\right) - u'\left(\tilde{c}_2\right)\right)\varepsilon(t)dt \end{aligned} \tag{H.4}$$

Appendix I. The representative agent assumption

Consider the following optimization problem:

$$U(X) = \max_{(x_i)_{i=1}^n \geq 0,} \sum_{i=1}^n u_i(x) \text{ s.t. } \sum_{i=1}^n x_i = X \tag{I.1}$$

Assume that $\forall i$, $u_i'(x) > 0$, $u_i''(x) > 0$, $\lim_{x \to 0} u_i'(x) = +\infty$, $\lim_{x \to +\infty} u_i'(x) = 0$ and let us show that $U(X)$ has the same properties. If λ denotes the Lagrangian multiplier, the first order necessary and sufficient optimality conditions are given by $\forall i = 1, \ldots, n$, $u_i'(x_i) = \lambda$ and $\sum_{i=1}^n x_i = X$. By solving this set of equations, we construct $(x_i(X))_{i=1}^n$ and $\lambda(X)$. It follows that:

$$U'(X) = \sum_{i=1}^n u_i'\left(x_i(X)\right) x_i'(X) = \lambda(X) \sum_{i=1}^n x_i'(X) = \lambda(X) > 0 \tag{I.2}$$

since $\sum_{i=1}^n x_i(X) = X$. So, $\sum_{i=1}^n x_i'(X) = 1$. Moreover, if we differentiate the first order conditions, we get:

$$\begin{cases} \forall i = 1, \ldots, n, \quad u_i''(x_i)dx_i = d\lambda \\ \sum_{i=1}^n dx_i = dX \end{cases} \tag{I.3}$$

$$\Leftrightarrow \begin{cases} \forall i = 1, \ldots, n, \quad \frac{dx_i}{dX} = \frac{1}{u_i''(x_i)} u_i''(x_i) \left(\sum_{i=1}^n \frac{1}{u_i''(x_i)}\right)^{-1} > 0 \\ \frac{d\lambda}{dX} = \left(\sum_{i=1}^n \frac{1}{u_i''(x_i)}\right)^{-1} < 0 \end{cases} \tag{I.4}$$

It follows that $U''(X) = \lambda'(X) < 0$. It remains to verify the Inada conditions. First, suppose that $X \to 0$, then, by the constraint, $\forall i$, $x_i \to 0$. As $\lim_{x \to 0} u_i'(x) = +\infty$, the first order conditions say that $\lambda \to +\infty$ and Eq.(I.2) leads to $\lim_{X \to 0} U'(X) = +\infty$. Now, suppose that $X \to +\infty$, then for at least one of i, $x_i \to +\infty$. As $\lim_{x \to +\infty} u_i'(x) = 0$, then, with the same argument, $\lambda \to 0$ and $\lim_{X \to +\infty} U'(X) = 0$.