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Conditioning Public Pensions on Health: Effects on Capital Accumulation and Welfare*

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Abstract

This paper develops an overlapping generations model which links a public health system to a pay-as-you-go (PAYG) pension system. It relies on two assumptions. First, the health system directly finances curative health spending on the elderly. Second, public pensions partially depend on health status during old age, by introducing a component which is indexed to society's average level of disability. This way, reducing disability during old age lowers the pension benefit as the need to finance long-term care services also drops. We then study the effects of introducing such a 'comprehensive' social security system on individual decisions, capital accumulation, and welfare. We first show that under certain conditions, health investments can boost savings and capital accumulation in the long run. Second, we show that if individuals are sufficiently concerned with their health when old, it is optimal to introduce a health-dependant pension system, as this will raise social welfare compared to a system where pensions are not tied to the society's average level of old-age disability. Our analysis thus highlights an important policy recommendation: making PAYG pension schemes partially health-dependent can be beneficial to society.

JEL Classification: H55, I15, O41.

Keywords: Curative Health Investments, PAYG Pension System, Disability, Overlapping Generations, Long-term Care.

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1 Introduction

Because of impressive health care improvements over the last fifty years, longevity has risen substantially in all developed countries. This has not come without challenges, however. More than two out of five individuals aged 65 or older report having some sort of functional limitation – ranging from sensory, physical, mental, or self-care disabilities – so that the importance of long-term care (LTC) has grown together with the number of the elderly (Siciliani, 2013).[1]

Since LTC is very costly, many possible avenues have been discussed over the years to guarantee sufficient financial resources to the elderly in case they effectively become dependent. A first solution could be to incentivize the purchase of private LTC insurance, but this has been shown to be relatively ineffective. Individuals often fail to insure themselves against the risk of becoming dependent at a later age, despite the substantial costs associated with the loss of autonomy. Many explanations have been suggested for this 'LTC insurance puzzle', both on the supply-side (e.g. adverse selection leading to high loading costs) and the demand-side (e.g. myopia, bequest motives) of the market.[2] A second solution would be to let the family take care of their elderly, through either formal (in cash) or informal (in time) care, but this may come at the expense of reduced labour supply of informal caregivers, higher psychological costs, and possibly lower aggregate welfare (De Donder and Leroux, 2017).

The last solution, and this is the one we explore in this paper, is to let the government step in and provide financial support to the elderly if they become dependent. More specifically, we propose a 'comprehensive' Social Security system that combines both a health system and a pension system which are closely interconnected. On the one hand, the government invests in public health so as to improve the quality of life of the elderly. On the other hand, pension benefits are augmented by a health-related component, indexed to the society's average level of old-age disability.[3]

[1]We follow Cremer (2014) in defining long-term care as: "the provision of assistance and services to people who, because of disabling illnesses or conditions, have limited ability to perform daily activities such as bathing and preparing meals." LTC is mainly targeted toward the elderly, with needs arising from various chronic diseases (mostly diabetes and -increasingly- cancer), Alzheimer or other forms of dementia.

[2]On the LTC insurance puzzle, see among others, Pauly (1990), Pestieau and Ponthière (2012), Brown and Finkelstein (2009), Brown and Finkelstein (2007), Lockwood (2018) and Boyer et al. (2020).

[3]In Québec in 2015, the implementation of an additional benefit to retirees above a given age was discussed in order to cope with the potential additional costs due to a loss of autonomy at older age (Hébert, 2016). Because of a change in government majority, this so-called *assurance autonomie* was never implemented.

The assumption we make is that reducing the intensity of dependency at old age will lower pension benefits as the need to finance long-term care services will also decrease. As a result, the government has a double interest in investing in public health. First, better health will increase individual welfare as it improves the *quality* of life during old age. Second, for a given average longevity, better health will decrease financial pressures on the social security system. This paper thus aims to show how public health investments targeted toward the elderly may be useful in designing more efficient pension schemes and in turn, in increasing aggregate welfare.

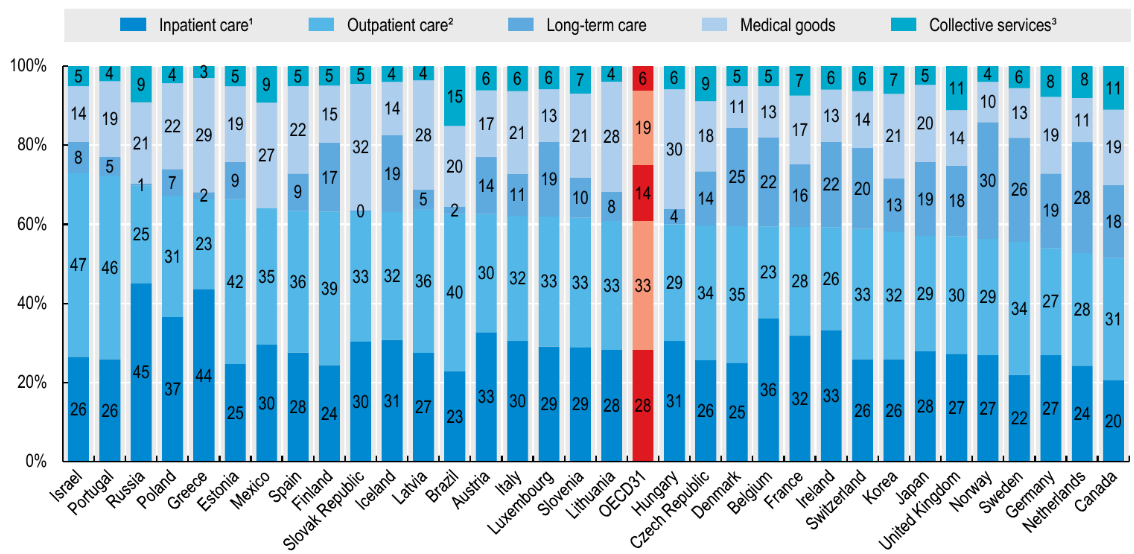
Our modeling of this augmented pension system then offers a more optimistic view of the future of social protection in general and, of pension systems in particular. Indeed, in the many countries facing the consequences of population aging, the public debate tends to disseminate the idea that health improvements are one of the factors behind unsustainable PAYG pension systems.[4] Conversely, our approach shows that by investing in public health and hence, improving the quality of life during old age, the need for additional LTC expenditures is mitigated. We show that this can, in turn, boost saving, nudge up wages and ultimately increase overall welfare. This will in particular be the case when the level of health taxation is not too high and the PAYG pension scheme is made sufficiently health-dependent.

Our objective is thus to better understand the effects of increasing public health investment in the quality of life during old-age on capital accumulation, growth and welfare when the PAYG public pension scheme partially depends on society's average level of old-age disability. To do so, we extend the textbook overlapping generations (OLG) model introduced by Diamond (1965) by allowing pension benefits to include both a universal component (the standard PAYG pension benefit) and a component that will be health-dependent (what we call the "disability-augmented" part of the pension benefit). Such a modulation of the pension system ties pension benefits to the health-status during old age, something which clearly influences the amount of resources needed when old, in particular if individuals become dependent and need to resort to LTC.

Since our aim is to study the effects of reducing the intensity of dependency during old age, we focus on *curative* health spending to improve the quality of life during old age. A few

[4] This is inherent to the design of PAYG pension systems, where working individuals pay for the pensions of retired individuals. If healthier people live longer there will logically be fewer working contributors for each pensioner, a downward trend which is projected to accelerate (Pecchenino and Pollard, 2005). See also Cigno and Werding (2007) on increasing age-dependency ratios.

examples are: building nursing homes, increasing the number of nurses and providing subsidies for LTC expenditures. In a recent report, the OECD (2021) indicates that a vast majority of public health spending is directed toward curative care rather than disease prevention and health promotion. In particular, curative and rehabilitative care services, account for 60% of total health spending in OECD countries. Moreover, Figure 1 (Figure 7.15 in the OECD report) also indicates that the majority of health spending is primarily provided through inpatient and outpatient treatments.



Note: Countries are ranked by curative and rehabilitative care as a share of current expenditure on health.
1. Refers to curative and rehabilitative care in inpatient and day care settings. 2. Includes home care and ancillary services. 3. Refers to prevention and administration and includes unknown or unallocated services.
Source: OECD Health Statistics 2021.

Figure 1: Health expenditure by type of service, 2019 (or nearest year),
Source: OECD (2021)

Our model assumes a representative individual lives two periods of fixed length with certainty. He derives utility from consumption in both periods and from being autonomous during old age. In the first period, he works, contributes to the health system and the Social Security system. He also saves for old-age consumption. In the second period, he consumes the proceeds of his savings and the pension benefit. Public intervention is twofold. First, and using the health contributions, the government invests in public health by making curative expenditures. Contributions of workers in a given period are directly invested to increase the health condition of the old in the same period. This directly increases the elderly' s utility since they value better health (equivalently, higher quality of life) during old age. Second, individuals contribute to a PAYG pension system in the first period and receive a pension benefit in the second one. As already mentioned, we will model an augmented version of a standard pension system by allowing the amount of the pension benefit to vary with the average level of disability in the economy. If health during old age deteriorates (equivalently, the intensity of dependency is higher in the society), the elderly will receive higher pension benefits so as to cope with extra LTC expenditures.

Before going further, let us note that, in this paper, we abstract from the potential impact of health expenditures on increased longevity, which in turn would affect the pension budget constraint. This way, we can exclusively concentrate on the consequences on the economy of an improved quality of life during old age, regardless of lifespan changes. To our knowledge, no other study considers the effect of curative (rather than preventive) health investments in explaining capital accumulation and welfare in presence of an augmented PAYG pension scheme.

In order to answer our research questions, we first start by computing the steady-state level of capital, as a function of health and pensions contribution rates. Then, to analyze the problem in a context that would match reality more closely, we run a second-best analysis assuming that social security tax rates are close to those observed in OECD countries (and maybe sub-optimal) and look at how higher public health spending impacts health and capital accumulation. We finally study the welfare effects of introducing a disability-related component to the pension benefit. To do so, we fix a minimum PAYG pension contribution and find the optimal levels of both the public health tax and of the additional social security tax rate which would finance the disability-augmented part of the pension benefit. This way we can quantify the welfare increase resulting from implementing such an augmented pension system. Importantly, we relate the optimal tax rate levels and the welfare increase to the individuals preference for being healthier during old age (i.e. less disabled).

Our main results are twofold. First, we prove analytically and illustrate it numerically that, if the social security contribution rates are close to those observed in OECD countries, higher public health expenditures foster capital accumulation and growth, as long as the contribution rate to the health system is not too large. This is the result of two opposing forces. On the one hand, increasing the health tax increases average health (i.e. decreases average disability) so it decreases pension benefits in the second period, and thus increases the willingness to save for old age. On the other hand, a higher health tax decreases disposable income and thus, savings. If the health tax is not too big, the first effect dominates the second one: savings increase, and so does capital and output per capita.

The second contribution we make concerns the welfare effects of introducing a health-augmented pension benefit. We show that if individuals are sufficiently concerned with their health at the old age, it is optimal to introduce a health-augmented pension system and, it will increase social welfare in comparison to a system where pensions are not tied to society's average level of old-age disability. In addition, as the individual's preference for health increases, the welfare gains of augmenting the PAYG system become larger.

Relating these findings with what we observe in reality, it is reasonable to think that in the last decades, individuals have become more concerned with their health (in all stages of life) as a consequence of first, population aging (and with it, an increasing probability to become dependent) and second, of our economies becoming richer and more informed about health issues. Our results then prove that it would be indeed optimal to introduce such a health-augmented pension system under this new reality.

The paper proceeds as follows. Section 2 summarizes the existing literature. Section 3 presents the model, and establishes the equilibrium. Section 4 describes the effect of a rise in health taxation on steady-state capital accumulation. Section 5 combines our findings to shed light on the potential welfare ramifications brought about by the kind of mixed pension system we propose. Section 6 concludes.

2 Related Literature

Our paper can be related to at least two strands on the literature. First, it can be related to the vast literature on economic growth (see de la Croix and Michel, 2002) and on overlapping generations (OLG) models which have been building on the seminal work of Diamond (1965). OLG models are particularly suited to study the impact of major demographic changes over consumption, capital accumulation and growth as well as the effect of policy changes on these economic variables. In particular, Diamond's model has been extended to capture the effects of increased longevity, decreasing fertility and human capital formation on pensions (See e.g. Kaganovich and Meier (2012), Fanti and Gori (2012), Cipriani (2014), and Chen (2018)). Interestingly for our paper, Chakraborty (2004) extends the model of Diamond (1965) to allow for public health expenditures to endogenously determine longevity. He shows that raising taxes to finance public health investments then improves survival chances of the elderly. Anticipating a longer lifespan, agents choose to save more to smooth consumption at an older age, thereby boosting capital accumulation in the long run. Our paper differs from the strand of the literature which followed Chakraborty (2004) and assumed endogenous longevity, in at least two respects.[5] First, unlike this literature, we assume that (curative) public health expenditures affect the *quality* of life during old age, or equivalently the intensity of disability of the old age period itself or the individual's survival probability. The question we ask is to what extent the reduction in morbidity (due to higher health expenditures) fosters capital accumulation and growth. Second, we look at the interplay between public health programs and PAYG pension systems and, show how their joint design affects capital accumulation, growth and ultimately aggregate welfare, something which is absent from the existing OLG literature.[6]

Second, our paper can be related to the growing literature on rising LTC needs as a consequence of aging, and its impact on social welfare. To date, most of the existing literature has concentrated on the reasons why there exists a LTC insurance puzzle (see footnote 2). Most of the theoretical papers which rationalize the reasons underlying the absence of LTC insurance are partial equilibrium models.[7] The theoretical literature studying the general equilibrium impacts of rising LTC needs on capital accumulation and growth is still quite scarce. An exception is the contribution by Hemmi et al. (2007) which show how after-retirement health shocks affect the level of precautionary savings and how multiple steady-state equilibria may exist depending on the level of economic development. They assume away any Social Security program. Kopecky and Koreshkova (2014) also calibrate a life-cycle model where individuals face risk on earnings, survival and LTC expenses, but can partially insure through public programs such as Medicaid and Social Security. They show that a sizeable part of wealth is devoted to planning for old-age risks of survival and of needing LTC. They

[5] See for instance, Blackburn and Cipriani (2002), Bhattacharya and Qiao (2007), de la Croix and Ponthiere (2010), Jouvet et al. (2010), de la Croix and Licandro (2013), Fanti and Gori (2014), Zhao (2014) and Davila and Leroux (2015) which assume endogenous lifetime.

[6] One exception is Grossmann and Strulik (2017) which studies the effect of medical progress on health spending, public savings for old age and the retirement age. They show that increasing medical progress is in conflict with reducing health inequalities. Contrary to us, they have no private savings and the pension system does not include the disability component.

[7] See e.g. Pestieau and Sato (2008),Canta and Pestieau (2013), De Donder and Leroux (2014), De Donder and Leroux (2017) and De Donder and Pestieau (2017).

further show that all young generation would benefit from making these public programs more generous. Finally, Canta et al. (2016) study the dynamic of capital accumulation when individuals face a probability to become dependent during old age and when the existence of family norms can influence the level of informal care provided by the family. They allow for a social insurance over the LTC risk but take it as given.

Differently from to these papers, we model a public pension system which is closely linked to the level of public health investment and, more specifically, which allows to account for the fact that if the health at old age deteriorates (resp. improves), this should be reflected in the level of the pension benefits to be distributed. As such, we claim that both public programs should not be considered in isolation, contrary to what is mostly assumed in the above cited literature. Furthermore, we provide a normative analysis and show that introducing a health component in the pension benefit formula is welfare improving.

Shedding some light on the long-term implications of combining the health care system with the pension system, we aim to cover several blind spots in the policy debate as well.

3 The Model

We consider a closed economy populated with perfectly foresighted and identical individuals whose lifespan is finite and divided into two periods: youth (the working period), and old age (the retirement period). We assume no uncertainty on lifespan and each period duration is normalised to one.

At every date t, a mass N_t of newly born individuals (generation) overlaps with the previous one. N_t grows at an exogenous rate of $n \in (-1; +\infty)$, so that $N_t = (1+n)N_{t-1}$.

When young, the representative agent born at t inelastically supplies one unit of labor, and earns the competitive wage rate w_t. When retiring, agents consume their accumulated savings as well as the pension benefits (which we formally define below) provided by the government.

The public system consists of two separate pillars: a pension system and a health care system. These two pillars are detailed below.

3.1 Public Health Investment

We assume that individuals have perfect health (normalised to 1) in the first period of their life while in the second (old age) period, their health has deteriorated. Their health status at the old age, or equivalently their level of autonomy, is denoted by d_t, and it depends on the public health investment in quality of life at the old age (denoted by h_t) as in the following equation:

$$d_t(h_t) = \frac{h_t}{1+h_t}. \tag{1}$$

In particular d_t is an increasing and concave function of h_t with $d(0) = 0$, $\lim_{h\to\infty} d(h) = 1$ and $\lim_{h\to 0} d^{'}(h) = 1$.

Quite importantly, observe that the time index of h_t in equation 1 is the same as the time index of d_t so that health expenditures are made in the same period as the benefits on health are experienced.[8] This happens because the objective of our paper is to consider curative rather than preventive health expenditures, as is the case for public health investment in quality of life during old-age. As already discussed in the introduction, such investments range from building nursing homes to providing day-to-day care, medications and therapies, or quite simply extending existing medical services.[9]

In order to finance public investments in quality of health of the elderly generation h_t, we assume that the government levies a health tax $\tau_h \in [0, 1]$ on the labour income of the current working generation. For the sake of simplicity, we set the proportional tax on gross wages to be constant over time and, assuming that at each period the health care budget needs to be balanced, we have

$$h_t = \tau_h w_t. \tag{2}$$

The objective of this paper is to understand the effect of *public* health investments in quality of life at the old age, on the substainability of pension systems, on capital accumulation and on welfare. For this reason, we voluntarily abstract from modeling private investments in health.[10] Our approach allow us to concentrate on the following research question: Which

[8] This specification is used by Chakraborty (2004) for the survival probability. A translated version of this function is also used by Blackburn and Cipriani (2002) or by Fanti and Gori (2014).

[9] Although the literature has already modeled the impact of preventive health investment in OLG models with PAYGO pensions (see Chakraborty, 2004 or Fanti and Gori, 2011a), to the best of our knowledge, no model has studied the effect of curative health investments in such a framework.

[10] In reality, the individual health condition, even at the old age, depends on many factors, such as for example, personal behaviour and lifestyle, as well as health investments (see among others, Balia and Jones (2008),

are the main economic impacts of considering curative rather than preventive public health investments in the standard Diamond's model with PAYG?

3.2 Introducing Health-Dependent Pensions

We assume a PAYG pension system so that young adults pay at period t for the pension benefits received by retirees in the same period t. In order to finance the pension system, a proportional contribution rate, $\tau_{p,t} \in [0,1]$ is paid by the workers. This contribution rate takes the following form:

$$\tau_{p,t} = \tau_0 + \tau_1(1 - d_t) \tag{3}$$

where $\tau_0 \in [0,1]$ and $\tau_1 \in [0,1]$ are exogenous constants. The contribution rate paid by workers is then twofold. The first part τ_0 does not depend on the current health status in society; the second one, $\tau_1(1 - d_t)$ does. This second (health-related) contribution part constitutes the main difference with respect to existing models of pensions and LTC, which always consider pensions and LTC systems as two separate systems, and never condition the taxes to be paid on the current average level of old-age disability in society. The idea here is to make workers pay more whenever the health status of the old in the society deteriorates (i.e. d_t is smaller).[11] For that reason, the contribution rate to the pension system depends on time t, through d_t (or h_t).

Assuming that the pension system is balanced at each time period t, each retiree will receive a pension benefit p_t of the following form:

$$p_t = (1 + n)(\tau_0 + \tau_1(1 - d_t))w_t \tag{4}$$

where the left-hand side accounts for all pension benefits given to retirees and the right-hand side accounts for total contributions to the pension system.

The first part in the above equation, i.e. $(1+n)\tau_0 w_t$, accounts for the *traditional* contribu-

Contoyannis and Jones (2004), Ettner (1996), Kaplan et al. (1987)). If we wanted to include private investments in health , we would have to make further assumptions regarding complementarity or substitutability between public and private health spending. This would complicate the solving of the model without further qualitative results.

[11] Indeed, this is comparable to the "Journée de la solidarité" (i.e. "solidarity day") which was implemented in France in 2004. This policy measure consisted in suppressing one vacation day so as to finance LTC at the old age and disability programs. This can equivalently be seen as imposing an additional contribution based on the average old-age disability level in the society.

tive part of the PAYG pension benefit. The second part i.e. $(1+n)\tau_1(1-d_t)w_t$, constitutes the novelty of our approach with respect to standard retirement models: it is the *'disability-augmented'* part of the pension benefit. If health were perfect (i.e. individuals remained fully autonomous at the old age), $d_t = 1$ and the second part would vanish. In that case, retirees would obtain a lump sum benefit, $p_t = \tau_0(1+n)w_t$, independent from the health status in society. Inversely, whenever health is imperfect, i.e. $d_t < 1$, workers face an additional tax contribution which is redistributed to retirees. The better (resp. the worse) the health condition at old age in society (through higher -resp. lower- public investment in health), the lower (resp. the higher) would be the tax rate and thus the pension benefits to be served. This additional benefit, indexed to the average level of disability in society, can then be used to finance the additional LTC expenditures caused by a lower level of autonomy during old age.[12]

3.3 Individuals

The lifetime welfare of the representative individual of generation t is modeled through a homothetic and separable utility function U_t, defined over consumption of a private good at the young and the old ages, and over the health status at the old age:

$$U_t = \ln(c_{1,t}) + \beta \ln(c_{2,t+1}) + \gamma \, d_{t+1}, \tag{5}$$

with $c_{1,t}$ and $c_{2,t+1}$, denoting private consumption respectively at the young age and at the old age (when retired), and d_{t+1} is the health status in old-age which, as we detailed in Section 3.1, depends on public health investment. The parameter $0 < \beta \leq 1$ accounts for time preferences, while $\gamma > 0$ is an individual preference for health parameter.[13] A higher γ corresponds to higher weight attributed to health during old age, and as we show below, it will motivate higher public investment in health and higher health-related tax rates. As a preference for desired health-status during old age, this parameter can also be thought of as a health-oriented expression of 'future orientation'.[14]

[12] We implicitly assume here that becoming dependent at the old age entails higher overall financial needs than remaining autonomous. On this see (De Donder and Leroux, 2021).

[13] This modeling of the preference for health is similar to Fonseca et al. (2020) and Hall and Jones (2007).

[14] In psychology and related fields, 'future orientation' is broadly defined as the extent to which an individual thinks about the future, anticipates future consequences, and plans ahead before acting. In a survey covering French respondents, Apouey (2018) explores this broader notion of time preferences over the future, and finds

Assuming separability is standard in the OLG and the health economics literature. Indeed we implicitly assume here that there is no complementarity or substituability between health (or disability) and consumption in old age, or equivalently that the marginal utility of old-age consumption does not depend on the health status (i.e. cross derivatives between consumptions and health are null). As argued by Hall and Jones (2007) and De Donder and Leroux (2021), whether substitutability and complementarity should be assumed is an empirical (and debatable) question. It crucially depends on the type of consumption goods considered, whether it includes traveling, food, restaurants or, to the opposite, medical expenditures and the adaptation of the house to limitations in daily-life activities. In the former case, consumption and good health are likely to be complements; in the latter case, they are likely to be substitutes. In order to keep our analysis tractable, we decided to abstract from these considerations.[15]

At time t, young individuals join the workforce and offer one unit of labour to firms, and receive the competitive wage w_t. This salary is taxed to finance both public health investment and the pension system. Therefore, the budget constraint of the young agent at time t is given by

$$c_{1,t} + s_t = w_t(1 - \tau_h - \tau_{p,t}). \tag{6}$$

where s_t are the individual's savings. Savings are deposited in a mutual fund accruing at a gross return of r_{t+1}. Note that the overall tax rate needs to be below unity: $(\tau_{p,t}+\tau_h) \in (0, 1]$.

During old age, consumption c_2 is financed out of savings and pension benefits. The budget constraint of an old agent born at time t then writes as

$$c_{2,t+1} = s_t(1 + r_{t+1}) + p_{t+1}, \tag{7}$$

with p_{t+1} the pension benefit as defined by (4). Substituting equations (4), (6) and (7) into (5) and maximizing the individual's utility function U_t w.r.t. savings, it can be shown that

correlational evidence linking it to preparation for old age (showing for example in saving behaviour, home adaptation and ownership, but *not* in LTC insurance). Similarly to Reckers-Droog et al. (2021), this preference parameter can then be seen as representing the marginal willingness to pay for health, but then specifically oriented towards future old age.

[15] In unreported simulations, we could show that removing separability between health and consumption in the individual's utility does not qualitatively change our results. Computations are available to the authors upon request.

the optimal saving decision s_t of an individual born in period t is equal to

$$s_t = \frac{(1+r_{t+1})w_t(1-\tau_h-\tau_{p,t})\beta-(1+n)\tau_{p,t+1}w_{t+1}}{(1+r_{t+1})(1+\beta)}. \tag{8}$$

The second term in the numerator represents the pension benefit, p_{t+1} the individual expects to receive at the old age. The higher the pension benefit, the smaller the amount of savings, s_t. Hence, if the government increases public health spending h_{t+1}, this will improve the health status at the old age (d_{t+1}), but at the same time it will decrease the amount of pension received. In turn, the individual will choose to save more.

3.4 Firms

At every period t, firms produce a final good whose price is normalised to unity, for an amount $Y_t = F(K_t, L_t)$, that can be consumed, saved or invested in public health. Since in our model, every young agent works, $L_t = N_t$ and assuming a standard Cobb Douglas production function, we have:

$$F(K_t, N_t) = AK_t^{\alpha}N_t^{1-\alpha} \tag{9}$$

with $\alpha \in (0,1)$ and where $A > 0$ accounts for the exogenous technology productivity or equivalently, total factor productivity.

We define the production function in *per capita* terms:

$$y_t = \frac{Y_t}{N_t} = f(k_t) = Ak_t^{\alpha},$$

with k_t is capital per unit of labor.

Perfect competition in the goods market implies that both capital and labor are paid at their respective marginal productivity, so that the wage is $w_t = \frac{\partial F(K_t,N_t)}{\partial N_t}$ and the the rental rate of capital is $R_t = \frac{\partial F(K_t,N_t)}{\partial K_t}$. Assuming (in the latter) full depreciation of capital we get

$$w_t = (1-\alpha)Ak_t^{\alpha} \qquad \text{and} \qquad R_t \equiv 1 + r_t = \alpha Ak_t^{\alpha-1}. \tag{10}$$

Again, in the situation of complete depreciation of the capital at each period, we obtain the following relationship between capital and savings:

$$k_{t+1} = \frac{s_t N_t}{N_{t+1}}. \tag{11}$$

3.5 Equilibrium

Given the initial level of capital stock k_0, the competitive equilibria are characterized by a sequence of $\{k_t\}$ that satisfy jointly the savings condition defined in (8), the full depreciation capital equation (11), the prices equation (10) and, the health status and health expenditures equations (1) and (2) respectively. A steady state equilibrium is an equilibrium in which the values of k_t (and thus, of all the other variables) are constant over time. The following proposition provides an existence and a uniqueness result for the steady state equilibria.

Proposition 1. *Suppose that $\tau_h + \tau_0 + \tau_1 < 1$. Then, for a given initial level of capital $k_0 > 0$, the dynamics of $\{k_t\}$ of the* per capita *stocks of capital over time is well defined. The system admits two steady states: the trivial steady state where $k^* = 0$ which is unstable and a positive asymptotically stable steady state $k^* > 0$.*

Proof. See Appendix A. □

For further use, we denote the steady state values of consumption in both periods as well as the health status associated to $k^* > 0$, by c_1^*, c_2^* and d^*.

4 Public Health Investment and Capital Accumulation

Having established the equilibrium, we now focus on the long-term welfare effects of public health investment when the pension system is made contingent to the old-age level of disability in society (that is, p_t depends on d_t). To do so, we first find the impact of exogeneously increasing τ_h on the steady-state level of capital k^*. This is crucial since the level of capital determines long-term outcomes such as wages, interest rates, and ultimately welfare (for the variation of welfare, see Section 5).

Such a comparative statics exercise is far from trivial. On the one hand, a higher health tax reduces disposable income, in turn discouraging savings and eroding capital stock. On the other hand, a higher health tax increases health at the old-age (through higher public health investments) so that the contribution rates to finance pensions decrease, which increases disposable income, increases the willingness to save and fosters capital accumulation. Nonetheless, Proposition 2 below shows that, if the health-related pension contribution rate, τ_1 is sufficiently high, the latter effect dominates. This will be true also as far as the health taxation rate τ_h is not too high and below some threshold $\bar{\tau}_h \in [0, 1]$ (defined in Appendix A) which itself depends on the specific values of τ_0, τ_1.

Proposition 2. *Assume an economy where the PAYG pension benefits depend on the average health status at the old age in the society. If τ_1 is sufficiently high, and if the health tax rate is below some threshold $\bar{\tau}_h \in [0, 1]$, an increase in public health investment through an increase in τ_h boosts the steady state stock of capital, k^*.*

Proof. See Appendix A. □

When health taxation is low, i.e. $\tau_h < \bar{\tau}_h$, we find that the steady state level of capital increases following an increase in τ_h. In that situation, the increase in the health tax rate decreases the pension contribution rate (this corresponds to the second part in eq. (3)) which in turn increases disposable income and the willingness to save for old age. This effect is found to dominate the direct negative impact of a higher τ_h on disposable income of the young. The overall effect is an increase in savings and thus, in capital accumulation k^*. This, in turn, results in higher wages, and into even higher health investment and a better health condition of the elderly. This sparks off an indirect general equilibrium feedback effect which further encourages saving, and fosters capital accumulation. As a result, the steady-state output per worker increases.[16]

However, this multiplier effect is only triggered under specific conditions. First, the health tax cannot be too high, i.e. $\tau_h < \bar{\tau}_h$. Above this threshold, the agent may be liquidity constrained and may not be able to increase further his savings. Second, the contribution rate $\tau_{p,t}$ must be sufficiently responsive to the level of disability at old age d_t, that is τ_1 must be sufficiently high. This second point is quite intuitive, as our main mechanism works through the health-dependent contribution to the pension system. If an increase in τ_h is not allowed to substantially reduce the health-dependant contribution rate (through higher d_t),

[16] A similar mechanism where health investment bears on economic growth can be found in Chakraborty (2004), Fanti and Gori (2011b) and Fanti and Gori (2014).

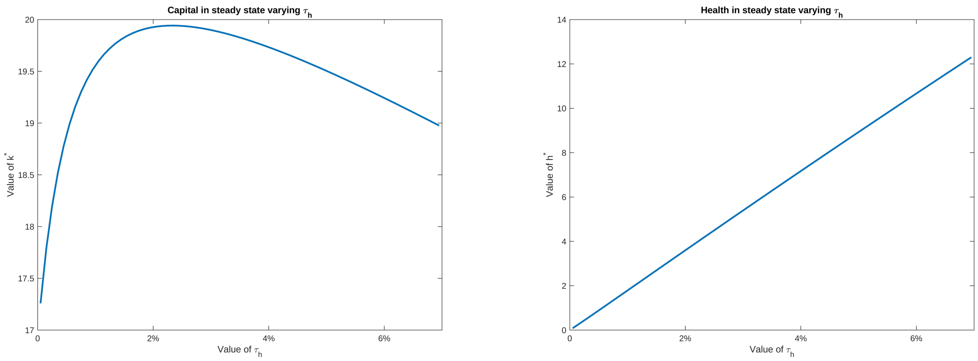


Figure 2: Numerical analysis: the effect of τ_h

,

we may well obtain that an increase in τ_h will decrease disposable income, savings and so, capital accumulation.

To illustrate how the steady state level of capital responds to an increase in the health tax rate, we perform a very simple numerical analysis in Figure 2. To this end, we set the parameter values at the following levels. First, we assume that each representative young adult individual (whose life starts at 20 years old) lives two periods of 40 years each. Assuming an annual discount rate equal to 1.25%, we find that $\beta = 0.6$, which is similar to the value of β in Žamac (2007).

We use a capital-output elasticity of $\alpha = 0.33$ as in de la Croix and Michel (2002). We then set the contribution rate at $\tau_0 = 8\%$ and $\tau_1 = 7.5\%$, because a majority of the OECD countries have rates lying in this range.[17] Following de la Croix and Michel (2002), we consider the replacement rate in a single-parent model over a period of 40 years, so that $(1 + 0.025)^{40} = 1 + n$, which gives an exogenous population growth rate of $n = 1.685$. Finally, we set the scale parameter A equal to 100.[18] We then perform a numerical exercise considering a range of health tax rate τ_h between 0% to 7%.

When τ_h is set to zero, the corresponding steady state level of capital is given by $k^* = 16.7$. Increasing the health tax, we find that the steady state level of capital increases up to 19.9 when $\tau_h = \bar{\tau}_h \simeq 2.3\%$, we find that increasing further public health taxation would reduce

[17]See for instance the graphs available at https://data.oecd.org/tax/social-security-contributions.htm. Over the period 2000-2020, the average Social Security contribution rates in the OECD countries have been around 8%, with France exhibiting rates as high as 14 and 16%.

[18]This level is admittedly a bit high. For instance, Chakraborty (2004) uses levels of 25 and 50. But, since this is a scale parameter, we decided to leave as it is.

capital accumulation. Figure 2 then confirms the theoretical results of Proposition 2.

All in all, this section shows that the observed positive relationship between increased public health expenditures and capital accumulation (when τ_h is not too high) is driven by the young individuals anticipating a better health status in old age and thus lower public pensions. Obviously, this would not occur under a standard (non-augmented) PAYG pension system. When $\tau_1 \to 0$, there is no effect of increasing health expenditures on pension benefits and thus, increasing public health taxation always reduces capital accumulation.

Finally, we find that the effect of τ_1 on the steady state level of capital is monotonic: increasing τ_1 always decreases capital accumulation k^* at the steady state.[19] To understand this, it is sufficient to note that savings and pension benefits are substitutes in transferring resources from the young age to the old age period. Increasing τ_1 (or equivalently τ_0) would unambiguously increase p_t which in turn would decrease the willingness to save for old age. In addition, higher first-period taxation would decrease disposable income and as such the saving capacity. These two effects go in the same direction as to decreasing savings and thus, capital accumulation in equilibrium.

5 Welfare

In this section, we first derive the optimal tax rates τ_1 and τ_h, taking τ_0 as given. We do so because in reality and for different historical and economic reasons (e.g. insurance motive, myopia) which we do not model here, public pension systems do exist. As such, because we start from a situation where a standard PAYG system already exists (i.e. $\tau_0 > 0$), we consider a *second-best* problem and show first to what extent the pension scheme should be augmented to take into account the disability-related component. Second, we quantify the welfare increase associated to augmenting the standard pension system.

5.1 Optimal taxation

In this section, we derive the second-best tax rate levels, τ_1^* and τ_h^*, postulating the existence of a pension system that would provide benefits during old age to the individual (as it is the case in most developed countries). To do so, we exogenously fix $\tau_0 \geq 0$ and we find the

[19] In order to prove this, we apply the implicit function theorem on k^* exactly like for the comparative statics with respect to τ_h (see proof of Proposition 2 in Appendix A.

values of τ_1 and τ_h that would maximize the utility of the representative agent at the steady state when varying the health preference parameter γ. More precisely, we study the impact of varying γ on the (non-trivial) steady state equilibrium characterized in Proposition 1.

Under our general equilibrium framework, choosing the optimal values for τ_1 and τ_h consists in maximising the utility of the representative agent, eq. (5), taking into account that c_1^*, c_2^* and d^* depend both directly on these tax rates as well as indirectly on them. Indeed, c_1^*, c_2^* and d^* (or h^*) depend on τ_1 and τ_h also through k^* in the pension benefits, wages and interest rates. Deriving analytically the first-order conditions of this problem with respect to τ_1 and τ_h proves to be tedious and would not provide interesting insights since we would obtain many different effects going in opposite directions.[20] This is why in the following we resort to numerical simulations and show how the optimal tax rates τ_1^* and τ_h^* vary with the preference parameter γ. We also consider different possible values of τ_0 ranging from 0% to 15%.[21] Our results are reported in Table 1.

Table 1: The effect of γ on tax rates

	$\tau_0 = 0\%$			$\tau_0 = 8\%$		
	τ_1^*	τ_h^*	$\tau_1^*(1-d^*)$	τ_1^*	τ_h^*	$\tau_1^*(1-d^*)$
$\gamma = 1$	0%	4.6%	0%	0%	4.5%	0%
$\gamma = 2$	0%	6.6%	0%	0%	6.5%	0%
$\gamma = 3$	0.9%	8.1%	0.1%	0%	8.0%	0%
$\gamma = 4$	3.0%	9.4%	0.1%	0%	9.2%	0%
$\gamma = 5$	4.7%	10.5%	0.2%	2.0%	10.3%	0.1%
$\gamma = 6$	6.1%	11.5%	0.2%	3.6%	11.3%	0.2%
$\gamma = 7$	7.3%	12.4%	0.3%	5.0%	12.2%	0.2%
$\gamma = 8$	8,3%	13.2%	0.3%	6.2%	13.0%	0.2%
$\gamma = 9$	9.3%	14.0%	0.4%	7.3%	13.8%	0.3%
$\gamma = 10$	10.1%	14.7%	0.4%	8.3%	14.5%	0.3%

	$\tau_0 = 5\%$			$\tau_0 = 15\%$		
	τ_1^*	τ_h^*	$\tau_1^*(1-d^*)$	τ_1^*	τ_h^*	$\tau_1^*(1-d^*)$
$\gamma = 1$	0%	4.5%	0%	0%	4.5%	0%
$\gamma = 2$	0%	6.5%	0%	0%	6.4%	0%
$\gamma = 3$	0%	8.0%	0%	0%	7.9%	0%
$\gamma = 4$	1.0%	9.2%	0.1%	0%	9.1%	0%
$\gamma = 5$	3.0%	10.4%	0.1%	0%	10.1%	0%
$\gamma = 6$	4.5%	11.4%	0.2%	1.7%	11.1%	0.1%
$\gamma = 7$	5.8%	12.2%	0.3%	3.2%	12.0%	0.2%
$\gamma = 8$	7.0%	13.1%	0.3%	4.5%	12.8%	0.2%
$\gamma = 9$	8.0%	13.8%	0.3%	5.7%	13.5%	0.3%
$\gamma = 10$	8.8%	14.6%	0.3%	6.7%	14.2%	0.3%

[20] This would become even more complex when deriving comparative statics with respect to γ.

[21] We use the same parametrization as in the previous section: $\alpha = 0.33$, $\beta = 0.6$, $A = 100$ and $n = 1.685$. In unreported simulations, we also find that the level of health d^* is always comprised between 0.90 and 0.96.

First note that the level of the contribution to the health system, τ_h^*, is always strictly positive, whatever the levels of τ_0 and τ_1^*. This is directly related to the fact that individuals value health in their utility function so that $h^* > 0$ is always optimal. On the contrary, τ_1^* becomes positive only above some γ threshold which will depend positively on the level of τ_0.[22] This is due to the fact that both τ_0 and τ_1 put pressure on financial resources available to the individual in the first period, so that when τ_0 increases, the minimum γ level for τ_1^* to be positive must also increase.

Let us now concentrate on the role of $\gamma > 0$ on the optimal pension design. An increase in the health-preference parameter always increases the optimal levels of the tax rates τ_1 and τ_h, for any given level of τ_0.[23] Indeed, when people exhibit higher preference for their health during old age, it is optimal for a government who seeks to maximise welfare to increase health-related taxes. Therefore, the optimal augmented-pension system crucially depends on the agents' relative preference for health during old age and the higher this preference, the higher should the health component in the disability-augmented pension system be.

All in all, this section shows that while it is always optimal to have a health system ($\tau_h^* > 0$), introducing a health component in the pension benefit (i.e. an augmented pension benefit with $\tau_1^* > 0$) becomes desirable only above some level of γ.

Relating these findings with what we observe in reality, it is reasonable to assume that in the recent years, individuals have become more concerned with their health (in all stages of life) as a consequence of first, population aging (and with it, an increasing probability to become dependent) and second, of our economies becoming richer and more informed about health issues. The above results then demonstrate that it would be indeed optimal to implement such a health-augmented pension system in this context.

5.2 Quantification of welfare

We now concentrate on quantifying the welfare effect of implementing an augmented pension system, that is the welfare effect of allowing the pension benefits to be conditioned on the average disability level in society. To do so, we set $\tau_0 = 8\%$ as in the average of OECD countries and we compare the welfare of the representative individual in steady state when

[22] Equivalently, the minimum γ level at which τ_1^* becomes positive increases with the level of τ_0.

[23] Note that the optimal tax rates increase nonetheless at a lower rate when γ becomes higher, which is due to the concavity with respect to consumption of the utility function.

$\tau_1 = 0$ (that is, when the pension system is a "traditional" one) with his welfare when $\tau_1^* > 0$ is set at its optimal steady state level (i.e. the pension benefit is "disability-augmented"). In both cases, τ_h is chosen optimally (i.e. as a solution of the individual's problem). We then quantify by how much society's welfare is increased when augmenting the standard PAYG pension system by its disability-related component. As we showed in the previous section, whether the optimal level of τ_1^* is strictly positive depends on the size of the health-preference parameter. In order to be able to compare welfare in the two pension situations (whether it is augmented or not), we will then concentrate on values of γ between 5 and 10 ensuring that $\tau_1^* > 0$.

Our results are reported in Table 2, which is divided in three parts. In the first four columns, we report the values of c_1^*, c_2^*, h^* (and τ_h^*) when τ_0 is fixed to 8%, τ_1 is exogenously fixed to 0% but τ_h maximises the steady state utility of the representative agent. In the second part of the table (columns six, seven, eight and nine), we report the values of c_1^*, c_2^*, h^* (and τ_h^*) when τ_0 is still fixed to 8% but both τ_1 and τ_h are (jointly) chosen to maximize the utility of the representative agent at the steady state.[24] The last column reports the relative variation in the utility between a standard PAYG system ($\tau_1 = 0\%$) and the augmented PAYG system ($\tau_1^* > 0$).

Table 2: The welfare effect of augmenting the PAYG system when $\tau_0 = 8\%$

	$\tau_1 = 0\%$				$\tau_1^* > 0$				% variation in U
	c_1^*	c_2^*	h^*	τ_h^*	c_1^*	c_2^*	h^*	τ_h^*	
$\gamma = 5$	87.6	297.5	0.947	10.2%	88.5	291.9	0.947	10.3%	+0.002%
$\gamma = 6$	85.5	299.0	0.950	11.1%	87.0	288.8	0.951	11.3%	+0.005%
$\gamma = 7$	83.6	300.5	0.954	11.9%	85.7	286.2	0.954	12.2%	+0.009%
$\gamma = 8$	81.9	301.8	0.956	12.7%	84.5	284.0	0.957	13.0%	+0.014%
$\gamma = 9$	80.3	303.0	0.958	13.4%	83.4	282.0	0.959	13.8%	+0.018%
$\gamma = 10$	78.8	304.1	0.960	14.0%	82.3	280.2	0.961	14.5%	+0.022%

Table 2 clearly shows that, given the existence of an universal PAYG pension system, augmenting the system with a health-dependent pension increases welfare when τ_1 and τ_h are set at their optimal levels and, provided that γ is sufficiently high. The effects of augmenting the pension system may look small when considering variations in utility, but this is due to the concavity of the log-utility function: for large values of c, the utility function is not very sensible to variations of c. Still, we can observe a relevant change in the consumption behavior

[24] We do not report the values of τ_1^* here, but these correspond to the values reported in the part of Table 1 where $\tau_0 = 8\%$.

of the agents.

6 Concluding Remarks

This paper has shown that introducing a social security system which combines both a public health system and an augmented-pension system (e.g. accounting for average old-age disability in the economy), can be beneficial.

To do so, we study an overlapping generations problem that includes both a health system and a PAYG pension scheme in which pension benefits are augmented by a health-related component, indexed to the average level of disability during old age. We show how such modification of the pension system affects capital accumulation and welfare.

Our analysis produces several insightful predictions regarding the effects of social security taxation on individual decisions, capital accumulation, and welfare. First, an increase in public health investment through an increase in the tax rate can boost the steady state stock of capital. This result emerges because health taxation improves health during old age, mitigating average disability in society, which in turn decreases pension benefits in the second period. This increases the willingness to save for old age as a result. Second, given the existing PAYG pension system, introducing a disability-linked pension component to the pension benefit allows for welfare improvements for the elderly. This will be the case if the preference for health in society is high enough. Third, the higher the individual preference for health in society, the larger the welfare gains of augmenting the PAYG system.

In a context where LTC expenditures are projected to rise dramatically in the near future, and where the society's preference for health has been increasing in the last decades, our model lays bare the importance of tying up public health investment against disability to public pension schemes as a way to increase society's welfare.

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A Proofs

Proof of Proposition 1. Using (8) and (10) one can rewrite (11) as

$$k_{t+1}(1+n)=\frac{(\alpha Ak_{t+1}^{\alpha-1})(1-\alpha)Ak_t^{\alpha}(1-\tau_h-\tau_{p,t+1})\beta-(1+n)\tau_{p,t+1}(1-\alpha)Ak_{t+1}^{\alpha}}{(\alpha Ak_{t+1}^{\alpha-1})(1+\beta)}$$

where $\tau_{p,t+1}$ is defined by (3). Replacing further for equations (1) and (2), we obtain after some manipulations,

$$\begin{aligned}k_{t+1}(1+n)(1+\beta)+\frac{(1-\alpha)k_{t+1}(1+n)(\tau_0+\tau_1+Ak_{t+1}^{\alpha}\tau_0\tau_h(1-\alpha))}{\alpha+Ak_{t+1}^{\alpha}\tau_h(1-\alpha)\alpha k_{t+1}^{\alpha}}&\\ =(1-\alpha)\left(1-\frac{(\tau_0+\tau_1+\tau_h+Ak_t^{\alpha}\tau_h(\tau_0+\tau_h)(1-\alpha))}{1+Ak_t^{\alpha}\tau_h(1-\alpha)}\right)Ak_t^{\alpha}\beta.&\quad (12)\end{aligned}$$

Denoting by $G(k_{t+1})$ (respectively, $F(k_t)$) the left- (resp. right-) hand side of the above condition, we need to have

$$G(k_{t+1})=F(k_t). \quad (13)$$

F and G are two functions that map $\mathbb{R}_+$ in $\mathbb{R}_+$ (for F is obvious while for G, we need to use the hypothesis $\tau_1+\tau_0+\tau_h<1$).

If we compute its derivative, we can easily see that $G'>0$ and $\lim_{k\to+\infty}G(k)=+\infty$, moreover $G(0)=0$ so for any choice of k_t there exists a unique k_{t+1} which satisfies the (13). This proves that, given an initial level of capital $k_0>0$, equation (13) uniquely defines a sequence of $\{k_t\}$ of positive *per capita* stocks of capital over time.

The values of k_t in steady state correspond to the k^* satisfying $G(k^*)=F(k^*)$.

We can observe that $F(0)=G(0)=0$ so in particular 0 is a steady state. Moreover (since $1-\tau_h-\tau_1-\tau_0>0$) $\lim_{k\to 0}F'(k)=+\infty$, $\lim_{k\to 0}G'(k)=\frac{\alpha(1+\beta)+(1+n)(\tau_0+\tau_1)(1-\alpha)}{\beta\alpha}\in(0,+\infty)$, that $\lim_{k\to+\infty}F'(k)=0$, $\lim_{k\to+\infty}G'(k)=\frac{\alpha(1+\beta)+(1+n)(\tau_0+\tau_1)(1-\alpha)(1-d_1)}{\beta\alpha}\in(0,+\infty)$ so $F(k)>G(k)$ for small k (different from 0) and $F(k)<G(k)$ for large values of k so there exists at least a solution of $F(k)=G(k)$ i.e. a steady state. To prove the uniqueness of strictly positive steady state one can observe that, after some computation, the condition $F(k)=G(k)$ for $k>0$ is equivalent to

$$\frac{(1+n)}{\beta A}k^{1-\alpha}=\frac{\left[(1-\tau_0-\tau_1-\tau_h)+Ak^{\alpha}\tau_h(1-\tau_0-\tau_h)\right]\alpha(1-\alpha)}{\tau_0+\tau_1+\alpha(1+\beta-\tau_0-\tau_1)+Ak^{\alpha}\tau_h(1-\alpha)(\tau_0(1-\alpha)+\alpha(1+\beta))}$$

after taking the power $\frac{1}{1-\alpha}$ of both side of this expression one can verify that the function on the left hand side is linear while the function on the right hand side (after some computation) is concave so the strictly positive steady state is unique. We can denote it by k^*. Since $G(k) > F(k)$ for all $k > k^*$ and $G(k) < F(k)$ for all $0 < k < k^*$ then k^* is globally asymptotically stable (and the null steady state is unstable).

□

Proof of Proposition 2. The unique strictly positive steady state k (see Proposition 1) is defined by the equation $G(k) = F(k)$ or, equivalently, $0 = H(k)$ with $H(k) := G(k) - F(k)$. Using the implicit function theorem, we have that $\frac{dk}{d\tau_h} = -\left(\frac{\partial H}{\partial k}\right)^{-1}\left(\frac{\partial H}{\partial \tau_h}\right)$. The latter can be computed explicitly and we obtain:

$$\frac{dk}{d\tau_h} = -(1-\alpha)^2 A k^{1+\alpha} \frac{I_1}{I_2}$$

where

$$I_1 = -k(1+n)\tau_1(1-\alpha) - \alpha\beta - Ak^{\alpha}(\tau_1 - 2\tau_h)(1-\alpha)\alpha\beta + A^2k^{2\alpha}\tau_h^2(1-\alpha)^2\alpha\beta$$

and

$$\begin{aligned} I_2 = (1+n)(1-\alpha)\left[-1 + Ak^{\alpha}\tau_h(1-\alpha)\right]^2 \alpha(1+\beta)k + k(1+n)(\tau_0+\tau_1) \\ + Ak^{1+\alpha}(1+n)\tau_h(1-\alpha)(2\tau_0 + (1-\alpha)\tau_1) + A^2k^{1+2\alpha}(1+n)\tau_0\tau_h^2(1-\alpha)^2 \\ - Ak^{\alpha}(1-\tau_0-\tau_1-\tau_h)\alpha^2\beta - 2A^2k^{2\alpha}\tau_h(1-\tau_0-\tau_h)(1-\alpha)\alpha^2\beta \\ - A^3k^{3\alpha}\tau_h^2(1-\tau_0-\tau_h)(1-\alpha)^2\alpha^2\beta \end{aligned} \quad (14)$$

When τ_1 is high enough (observe that, once other parameters are fixed, k is uniformly bounded as a function of τ_1 and τ_h), one can easily verify that the expression of $\frac{dk}{d\tau_h}$ is strictly positive for $\tau_h = 0$. By continuity, this is also the case for small values of τ_h. So in particular there exists $\bar{\tau}_h$ such that, once we fix A, α, ρ, β, n, γ, τ_0 and τ_1, k^* is an increasing function of τ_h on $[0, \bar{\tau}_h]$. This threshold level $\bar{\tau}_h$ is implicitly defined by

$$\frac{dk}{d\tau_h} = -(1-\alpha)^2 A k^{1+\alpha} \frac{I_1}{I_2} = 0$$

or equivalently by setting $I_1 = 0$. $\square$