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RATIFICATION AND VETO CONSTRAINTS IN MECHANISM DESIGN

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Working Paper GAEL ; 2006-09

- August 2006 -

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Ratification and Veto Constraints in Mechanism Design[1]

by

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This version: August 2006

Abstract

This note points out some restrictions imposed by the notion of ratification. This notion is widely used in the mechanism design literature that assumes that each agent has a veto power. We exhibit allocations that are not ratifiable and nevertheless can be implemented through a mechanism that gives veto power to the agents.

Keywords: Mechanism Design, participation constraints, ratifiability, veto.

JEL: C72, D78, D82.

[1]I wish to thank Francis Bloch and David Martimort for helpful comments. All remaining errors are my own.

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1 Introduction

This paper investigates the issue of voluntary participation to a contractual relationship and the impact of agents' veto power on contractual outcomes. We consider situations in which a mechanism designer (or mediator) offers a contract to a set of agents that all have veto power, i.e. situations in which the contract must be accepted by every agent to become binding. Such situations arise in a wide range of applied mechanism design models including models of pretrial negotiations (see Nalebuff, 1987), cartel agreements (see Cramton and Palfrey, 1990) or collusion in auctions (see Caillaud and Jehiel, 1998). More recently, a fastly growing literature on collusion in organizations (Laffont-Martimort, 1997 and 2000, Mookherjee and Tsumagari, 2004, Che and Kim, 2006, Jeon and Menicucci, 2006, Pavlov, 2006) also uses models of veto constrained mechanism design. In all those papers, a mediator offers to the agents a mechanism (the negotiated settlement, the cartel agreement or the collusive side-contract) that must be accepted by every one to become binding. A key issue is therefore the determination of the outside options. Indeed, in those models, reservation utility is not exogenously fixed but comes from the outcome of non-cooperative behavior in the strategic underlying game (the trial game, the product competition game, the auction or the grand-mechanism that governs the organization).

On a theoretical ground, a general treatment of veto power in mechanism design has been offered by Cramton and Palfrey (1995) who introduced the notion of *ratifiability*. Starting from a one-stage game in strategic form, they model the mechanism design problem as a three stage game. First, the mediator proposes a mechanism. Then each agent can accept or refuse this mechanism ; this is the ratification stage. Finally agents either play the mechanism if it was ratified, or play non-cooperatively the underlying game with updated beliefs if it was vetoed. [1]

All papers in the veto constrained mechanism design literature use this ratification approach and solve the models by computing the optimal ratifiable mechanism, that is to say the optimal mechanism that is accepted by every agent in the second stage. In particular in the collusion in organizations literature, the collusive side-contract is constrained to be ratifiable and by induction, the optimal design of the overall organization is constrained to be robust to ratifiable side-contracts.

In this paper, we argue that the ratification approach might not be as general as

[1] In the original *ratifiability* notion, there is a restriction on out-of-equilibrium beliefs. They are supposed to satisfy the perfect sequential equilibrium refinement. In this paper, we abstract from the belief refinement issue and concentrate on the sequence of events, in particular on the existence of a distinct ratification stage.

desired for the above mentionned applications and that imposing ratifiability is distinct from imposing veto constraints. We show that, starting from a one-stage game in strategic form, there exist some allocations that can be achieved via a mechanism that guarantees veto power but that cannot be achieved if we impose ratifiability. Therefore, the optimal veto constrained allocation may not be ratifiable. Our construction mixes the two roles of a mediator that were pushed forward in the literature (see Myerson, 1994): proposing binding contracts and proposing mediated communication protocols.

The possible loss of generality comes from the assumption that there is a distinct ratification stage in the ratifiable mechanism design timing. If we drop that assumption and assume that acceptance/refusal and type revealing messages are sent simultaneously by the agents, it is possible to enlarge the set of implementable allocations.

The note is organized as follows. In section 2 we detail the ratifiability and the veto constrained implementability notions. Then, we prove in section 3 that the set of veto constrained implementable outcomes is larger than the set of ratifiable outcomes and strictly larger for a non-empty set of underlying games. Section 4 concludes.

2 Veto constrained mechanism design

The starting point of the analysis is a strategic situation modelled by a one-stage N-player (or agent) game in strategic form G. As is usual in quasi-linear mechanism design problems, G can be a game of imperfect information and we assume that utility is transferable among agents. In the sequel, G is characterized by an information structure $\{(\Theta_i)_{i\in N}, (p_i)_{i\in N}\}$, and a payoff function for agent i, $U_i(\theta, s)$. With these notations Θ_i is the set of possible types for agent i, $p_i(\theta_{-i} \mid \theta_i)$ is the probability that a θ_i-agent i assignes to the state of nature $\theta = (\theta_i, \theta_{-i})$ and $s = (s_1, ..., s_n)$ is the vector of strategies followed by the agents. The functions p_i therefore characterize the beliefs of the agents. This game describes the environment of the agents if there were no contractual oppportunities. Introducing such opportunities transforms the initial game G into a multistage game. In the first stage, a mediator proposes a mechanism M to the agents which can specify the strategies s_i to be followed in the game G as well as monetary transfers t_i made to the agent as functions of the messages m_i received by the mediator. This mechanism induces a game which is subsequently played by the agents. The basic question is then, starting from game G, what final allocation can be reached through a contractual agreement. Throughout the analysis, we assume that agents have the ability to veto the proposed mechanism. Therefore, we impose in the following that, if at least one of the agents refuses to participate in the mechanism M, they cannot use a binding agreement to play G.

A possible formulation of our problem which takes into account the veto power of the agents is given below. Starting from G, consider the enriched game $\mathcal{G}$ which is described by the following timing:

• $\mathbf{t} = 1$ The mediator proposes a mechanism M,

• $\mathbf{t} = 2$ Agents simultaneously accept or refuse M. If at least one agent refuses M, then go to $\mathbf{t} = 3'$; otherwise go to $\mathbf{t} = 3$.

• $\mathbf{t} = 3$ Agents learn that M has been accepted by all. They send messages m to the mediator who implements $\{s(m), t(m)\}$ in G. The game ends.

• $\mathbf{t} = 3'$ Agents learn that M has been vetoed by at least one agent. Each agent updates its beliefs about the other agents' types and play non-cooperatively the game G with updated beliefs and mediated communication. The game ends.

The formulation detailed above takes into account the commitment power of the mediator and assumes that the communication protocol used at date $\mathbf{t} = 3'$ is included in the mechanism M announced at date $\mathbf{t} = 1$.

Definition 1 Ratifiability *A final allocation is ratifiable if and only if there exists a Perfect Bayesian Equilibrium of* $\mathcal{G}$ *that implements it.*

A justification for the term ratifiability stems from the fact that it can be shown that a ratifiable allocation can be implemented by a mechanism M that is accepted by every agent (see the proof of Proposition 1). Actually, Cramton and Palfrey (1995) use the term ratifiability to characterize mechanisms M and not allocations. The two uses are equivalent.

Veto power is taken into account by assuming, at date $\mathbf{t} = 2$, a ratification stage in the formulation of the mechanism design sequence of events. To the best of our knowledge, such a ratification stage is a common feature of all papers in the mechanism design literature that assume veto power. However, this concept of ratifiability is a slight generalization of that introduced by Cramton and Palfrey (1995) who do not explicitly consider the possibility of mediated communication. At date $\mathbf{t} = 3'$ they assumed that agents play a non-cooperative equilibrium of G played with updated beliefs whereas we asssume that they play a communication equilibrium of G played with updated beliefs, i.e. we assume that even if M is not ratified, the mediator can still be used to coordinate the strategies in G (therefore we follow Myerson (1994) and consider two possible roles for a mediator: offering binding contracts and offering mediated communication protocols).

In addition, they introduce some refinement, based on perfect sequential equilibrium, on out-of-equilibrium beliefs. We voluntarily abstract from belief refinement issues and concentrate on the sequence of moves to simplify the exposition of the argument which could be adapted if we impose some restriction on out-of-equilibrium beliefs. Of course, considering a stricter concept for ratifiability would only reinforce the results of Propositions 1 and 2.

Alternatively, consider the enriched game $\mathcal{G}'$ which is described by the following timing:

- $\mathbf{t} = 1$ The mediator proposes a mechanism M,
- $\mathbf{t} = 2$ Agents simultaneously accept or refuse M and send messages m to the mediator. If at least one agent refuses M, then go to $\mathbf{t} = 3'$; otherwise go to $\mathbf{t} = 3$.
- $\mathbf{t} = 3$ The mediator implements $\{s(m), t(m)\}$ in G. The game ends.
- $\mathbf{t} = 3'$ Agents learn that M has been vetoed. Each agent updates its beliefs about the other agents' types and play non-cooperatively the game G with updated beliefs and mediated communication. The game ends.

The difference between $\mathcal{G}$ and $\mathcal{G}'$ concerns the date at which messages are sent to the mediator. This is important to determine what occurs in case we reach date $\mathbf{t} = 3'$ because the information possessed by the mediator is not the same in the two games.

Definition 2 Veto constrained implementability *A final allocation is veto constrained implementable if and only if there exists a Perfect Bayesian Equilibrium of $\mathcal{G}'$ that implements it.*

3 Ratifiability is a restrictive notion

Moving from $\mathcal{G}$ to $\mathcal{G}'$, the mediator is endowed with more design abilities because he is not restricted to receive only acceptance/refusal messages at date $\mathbf{t} = 2$. Therefore, and as intuition suggests, we start by showing that the veto constrained implementability concept is, at least weakly, less restrictive than the ratifiability concept.

Proposition 1 *For any game G, the set of ratifiable allocations is included in the set of veto constrained implementable allocations.*

Proof: Consider any ratifiable allocation and the corresponding mechanism M that is offered by the mediator. According to $M = \{(a,r)_i, (m)_i, s(m), t(m), (m')_i, s'(m')\}$, agents can accept ($a$) or refuse ($r$) the mechanism at date $\mathbf{t} = 2$. If they all accepted it, they are required to send a message m_i to the mediator who then implements $(s(m), t(m))$ in G. If M is vetoed by at least one agent, agents send messages m'_i to the mediator who then sends back the message $s'_i(m')$ to agent i. Such a message s'_i is basically a recommended strategy and $s'(m')$ is designed to be directly truthful revealing and obedient (this is a direct consequence of the revelation principle for Bayesian games). For the agents, a (possibly mixed) strategy in $\mathcal{G}$ is described by $\Sigma_i(\theta_i) = \{(a,r), p_i(a), m_i(a), p_i(v), m'_i(v), s_i(s',v)\}$ where (a,r) is the decision to accept or refuse M. The function $p_i(a)$ describes the updated beliefs of the agent if M is accepted by everybody while $p_i(v)$ describes the beliefs if M has been vetoed. Then $m_i(a)$ is the message sent to the mediator at date $\mathbf{t} = 3$ and $m'_i(v)$, $s_i(s',v)$ describe the strategy in the communication game at date $\mathbf{t} = 3'$. In equilibrium, the strategies Σ_i are mutual best-responses and beliefs p_i are updated using Bayes rule whenever it is possible. In such an equilibrium, the payoff of a θ_i-agent i is

$$\Pi_i(\theta_i) = E_{\theta_{-i}|\theta_i} E_{\Sigma_i(\theta_i)\times\Sigma_{-i}(\theta_{-i})}[U_i(\theta, s_i, s_{-i}) + t_i(m_i, m_{-i})],$$

where $E_{\cdot}$ denotes the expectation operator. Because $\Sigma_i(\theta_i)$ is an equilibrium strategy, a θ_i-agent i is better off by playing $\Sigma_i(\theta_i)$ rather than $\Sigma_i(\tilde{\theta}_i)$:

$$\Pi_i(\theta_i) \geq E_{\theta_{-i}|\theta_i} E_{\Sigma_i(\tilde{\theta}_i)\times\Sigma_{-i}(\theta_{-i})}[U_i(\theta, s_i, s_{-i}) + t_i(m_i, m_{-i})],$$

and is also (at least weakly) better off than if he decides to veto M with probability 1.

We sart by showing that M is equivalent to a mechanism M^* that is accepted everywhere on the equilibrium path.

The mediator can always replicate via an accepted mechanism what is achieved via a communication game that follows a vetoed mechanism. Therefore it is possible to build an incentive compatible direct mechanism $M^* = \{(a,r)_i, (\theta_i)_i, s^*(\theta), t^*(\theta), (\theta_i)_i, s'(\theta)\}$ with

$$E_{\Sigma_i(\theta_i)\times\Sigma_{-i}(\theta_{-i})} U_i(\theta, s_i, s_{-i}) = U_i(s^*_i(\theta_i), s^*_{-i}(\theta_{-i})),$$

where s^* is possibly a mixed strategy and

$$E_{\Sigma_i(\theta_i)\times\Sigma_{-i}(\theta_{-i})} t_i(m_i, m_{-i}) = t^*_i(\theta_i, \theta_{-i}).$$

Such a mechanism M^* replicates the ratifiable allocation if it is accepted by all players. Following a veto, M^* specifies an incentive compatible and obedient direct revealing communication protocol, that exactly replicates what followed a veto in M:

$$E_{\Sigma_i(\theta_i)\times\Sigma_{-i}(\theta_{-i})}[U_i(s_i, s_{-i}) \mid r_i] = U_i(s'_i(\theta_i), s'_{-i}(\theta_{-i})),$$

where s_i' is possibly a mixed strategy. Acceptance M^* is therefore sustained by the out-of-equilibrium beliefs $(p_i(v))$ defined in the equilibrium strategies of M.

Consider now the game $\mathcal{G}'$ and the mechanism $M' = \{(\theta_i, r)_i, s^*(\theta), t^*(\theta), (\theta_i)_i, s'(\theta)\}$ that replicates M^* except that now, the message previously sent at date $\mathbf{t} = 3$ is sent at date $\mathbf{t} = 2$, simultaneously with the acceptance or refusal decision. On the equilibrium path agents take their decisions with exactly the same beliefs because Bayes rule impose that they do not change their beliefs after observing acceptance of the mechanism in M^*. Therefore M' is incentive compatible because M^* is. Out-of-equilibrium, i.e. after a veto, $p_i(v)$ can describe the agents' beliefs and the communication system $\{(\theta_i)_i, s'(\theta)\}$ is incentive compatible and obedient. Therefore, M' implements in $\mathcal{G}'$ the initial ratifiable allocation and this shows that the set of ratifiable allocations is included in the set of veto constrained implementable allocations. $\square$

We then show that in some situations it is strictly less restrictive.

Proposition 2 *For some games G, there exist allocations that are veto constrained implementable but are not ratifiable.*

Proof: To show that for some games G, the inclusion is strict, let us consider the following game G_1 depicted in Figure 1.

	l	c	r
u	$(3;3)$	$(1;1)$	$(0;1)$
d	$(-1;-1)$	$(-1;0)$	$(0;0)$

$\underline{\theta}$

	l	c	r
u	$(0;1)$	$(3;3)$	$(3;1)$
d	$(-1;-1)$	$(2;5/2)$	$(2;0)$

$\bar{\theta}$

Figure 1: Game G_1

In this game, agent 1 plays rows, agent 2 plays columns and the state of nature ($\underline{\theta}$ or $\bar{\theta}$) is agent 1's private information. We assume that θ is distributed according to

p =Prob($\theta = \underline{\theta}$) and $0 < p < 1/2$. We concentrate on allocations that maximize agent 1's payoff. Suppose that, in game $\mathcal{G}_1$, agent 2 vetoes the contract during the ratification stage. Then the two agents must play a communication equilibrium of G_1 played with passive beliefs (the beliefs agent 2 has on agent 1 cannot have changed because agent 1 played his equilibrium pooling strategy a, and agent 2 has no private information thus agent 1 cannot learn anything by observing a refusal by agent 2). Strategy u is dominant for a $\bar{\theta}$-agent 1 and the mediator cannot recommend another strategy for this type of agent 1 (otherwise agent 1 would not be obedient). Similarly, it can recommend to play strategy d to a $\underline{\theta}$-agent 1 only if agent 2 then plays r with probability 1 (this is implied by the obedience constraint) and must also play r in case the message sent by agent 1 is $\bar{\theta}$ otherwise a $\underline{\theta}$-agent 1 would have an incentive to lie (thus this second point is implied by the truthtelling constraints). But then, c strictly dominates r for player 2. We deduce from this reasoning that agent 1 can only be recommended to play u in all states. Then , because $p < 1/2$, agent 2 can guarantee himself a payoff equal to $3 - 2p > 2$ by playing c. We conclude that, if there is a ratification stage, the best expected payoff that agent 1 can obtain by contracting with agent 2 to play G_1 is $3 + 2p$ (the mechanism will implement $(u; l)$ or $(u; c)$ depending on the state of nature and a transfer equal to $2p$ in expectation).

Now, consider what can be achieved in $\mathcal{G}'_1$. The acceptance/refusal decision of each agent is incorporated in the messages simultaneously sent to the mediator at date $\mathbf{t} = 2$. The difference is that now, when agent 1 learns the refusal decision of agent 2, he already sent his information to the mediator : the out-of-equilibrium mechanism will not need to be truthful revealing and obedient, but only obedient. This will relax some important constraints in the case of G_1. Consider the following mechanism: agent 1 is asked to send an acceptance/refusal message together with his type information to the mediator and simultaneously, agent 2 is asked to send an acceptance/refusal decision to the mediator. In case both accept and agent 1 claims $\underline{\theta}$, the mediator implements $(u; l)$ and a transfer equal to 3 from agent 2 to agent 1; if both accept and agent 1 claims $\bar{\theta}$, the mediator implements $(u; c)$ and no transfer. If agent 2 refuses and agent 1 claims $\underline{\theta}$, the mediator recommends $(d; r)$ while he recommends $(u; c)$ in case agent 1 claims $\bar{\theta}$. This contract is accepted and truthfully played on the equilibrium path and brings a payoff equal to $3+3p$ to agent 1. Such an allocation is not ratifiable. $\square$

4 Conclusion

Building mechanism design models that give a veto power to each agent seems to be desirable for many applications ranging from pretrial settlement analysis to collusion study in auctions or in organizations. The standard and common approach used in the literature consists in introducing a distinct ratification stage in the mechanism design protocol to take into account the veto option. We showed that such an approach might involve some unsuspected loss of generality. There exist some veto constrained implementable allocations that are not ratifiable. In the paper we abstracted from belief refinement issues but they could be imposed on the veto constrained implementability notion as well as on the ratifiability notion without changing much of the argument.

References

Caillaud, B. and P. Jehiel, 1998, "Collusion in Auctions with Externalities," *RAND Journal of Economics*, 29: 680-702.

Che, Y.-K. and J. Kim., 2006, "Robustly Collusion-Proof Implementation," *Econometrica*, 74(4): 1063-1108.

Cramton, P. and T. Palfrey, 1990, "Cartel Enforcement with Uncertainty about Costs," *International Economic Review*, 31: 17-47.

Cramton, P. and T. Palfrey, 1995, "Ratifiable Mechanisms: Learning from Disagreement," *Games and Economic Behavior*, 10: 255-283.

Jeon, D.-S. and D. Menicucci, 2006, "Optimal Second-Degree Price Discrimination and Arbitrage: On the Role of Asymmetric Information Among Buyers," *RAND Journal of Economics*, 36(2):337-360.

Laffont, J.-J. and D. Martimort, 1997, "Collusion under Asymmetric Information," *Econometrica*, 65: 875-911.

Laffont, J.-J. and D. Martimort, 2000, "Mechanism Design under Collusion and Correlation," *Econometrica,* 68: 309-342.

Mookherjee, D. and M. Tsumagari, 2004, "The Organization of Supplier Networks: Effects of Delegation and Intermediation," *Econometrica*, 72(4): 1179-1219.

Myerson, R., 1994, "Communication, Correlated Equilibria and Incentive Compatibility," in *Hanbook of Game Theory with Economic Applications*, vol. 2, chap. 24, Elsevier North-Holland.

Nalebuff, B., 1987, "Credible Pretrial Negotiation," *RAND Journal of Economics*, 18(2): 198-211.

Pavlov, G., 2006, "Colluding on Participation Decisions," *mimeo.*